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RESEARCH MEMORANDUM

ATTITUDE SENSING AND CONTROL
FOR A SATELLITE VEHICLE

Robert E. Roberson
(North American Aviation, Inc.)

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The RAND *Corporation*

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REPORT

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MISSILE AND CONTROL EQUIPMENT

2 January 1953

ATTITUDE SENSING AND CONTROL

FOR A SATELLITE VEHICLE

By

Robert E. Roberson

NORTH AMERICAN AVIATION, INC.

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FOREWORD

The study reported here was performed for the Rand Corporation in accordance with the terms of their subcontract 52-34 of 1 March 1952, under USAF contract 33(038)-6413.

Its purpose has been the consideration of possible methods for the sensing and control of the attitude of a satellite vehicle, and the recommendation of the most promising systems for further theoretical and laboratory study.

This report embodies the efforts of a number of persons. The author wishes to acknowledge particularly his indebtedness to the following persons for aid, especially in connection with the chapters named: H. E. Singhaus for Chapters 2, 3, and 7; K. H. Rogers, B. P. Martin, and R. C. Sputhe for Chapters 2, 3, 7, and 9; A. F. Fairbanks and W. M. Cady for Chapter 6; D. L. Freebairn and F. J. Beutler for Chapters 4 and 8; K. P. Gow for Chapter 5; J. R. Conyers and R. S. Kraemer for Chapter 9. In few cases, however, have their contributions been confined to the chapters mentioned. Thanks are also due to all those, too numerous to mention specifically, whose discussions, helpful criticisms, and suggestions assisted in the prosecution of the study.

ABSTRACT

Possible methods of sensing and controlling the attitude perturbations of a satellite vehicle on orbit are discussed comparatively. The nature and magnitudes of torques causing the attitude perturbations are investigated, preliminary to selecting the most practicable systems. General design recommendations are made and topics worthy of further investigation are suggested.

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1. INTRODUCTION

1.1. PURPOSE OF STUDY

Vehicle and Orbit

The vehicle under consideration is an artificial satellite of the earth, circling the latter in a stable orbit with an average altitude above the surface of 560 to 800 km. As presently conceived, it is unmanned, has a gross mass on orbit of about 1000 kg, and carries a payload of about 450 kg. It is to be used for reconnaissance, televising pictures of the terrain over which it passes to receiving stations on the earth. The payload is devoted to equipment needed for the televising of information back to earth, for the placing of the vehicle on a stable orbit at the end of its trajectory phase, and for the maintenance of the vehicle in its proper attitude on the orbit.

If the ellipticity of the earth could be discounted, the desired vehicle orbit would be plane and circular, its center coincident with the center of the earth. The inclination of the orbital plane to the plane of the equator would determine the portion of the earth's surface under surveillance by the satellite. Thus, if the smaller dihedral angle between these planes were γ deg, all of the earth in the region from γ deg south latitude to γ deg north latitude generally would be seen in a period of time. The entire earth would be accessible only with a polar orbit (i.e., one following a meridian and crossing the poles). To realize these conditions, there would be a difficulty only in meeting the precision requirements on the measurement of vehicle attitude and speed at the transition point between trajectory and orbit. Small deviations from the required conditions would give an elliptical orbit with an ellipticity depending upon the deviation magnitudes.

Unfortunately, the ellipticity and magnetic field of the earth complicate this simple picture, and it can be shown that a circular orbit of a free satellite cannot be obtained by any means. The actual orbit is necessarily a space curve with continually changing curvature and torsion. Left to itself on such a path the satellite would present continually changing faces to the earth and to its forward direction. Similar changes in attitude arise from other perturbing effects, as discussed in Chapter 2. These variations are of great significance in the satisfactory use of the vehicle for reconnaissance, and are at the base of the attitude study reported here.

Vehicle Attitude

There are three questions which one may naturally ask concerning the vehicle attitude:

1. Why is attitude important to the desired use of the vehicle, and what attitude is required for this prospective operational use?
2. What causes deviations from the desired attitude?
3. How can attitude deviations be sensed and corrected?

The third is the central question of the study, and the second is answered in attacking the third. The first, partly answered below, motivates this study.

One importance of attitude is related to the problem of heat dissipation from the vehicle. The heat engine used to supply internal power to the vehicle during its orbital phase requires a heat sink. This will be provided by radiation into space from the skin of the vehicle. It is necessary for efficient design that this radiation from the surface be in a fairly constant direction relative to the earth. Thus, the orientation of the vehicle on its orbit must be controlled at least grossly.

A second, and much more stringent, attitude restriction is imposed by reconnaissance requirements. One of these is the necessity for directing the narrow-beam antenna for television transmission toward the ground receiving stations. Relatively complete attitude stabilization is desirable for this purpose, because direction instructions must be given relative to the vehicle structure. However, some deviation might be tolerated, and quantitative bounds can be placed only after the details of the transmission system are known. Also, it is desired to direct the telescope axis along a previously defined nadir direction. In scanning a band of terrain under the vehicle normal to its trajectory, frames will be taken obliquely to both port and starboard, as well as fore and aft, because the lateral scan is made, not instantaneously, but during a significant forward motion of the vehicle. Therefore, the condition placed on the nadir direction does not relieve the observer of adjusting each frame, but does insure that information is not lost at the edges of the scan because of excessive obliquity. Finally, a similar restriction holds on the yaw of the vehicle. If the nadir line is presumed correctly directed, then the angle between longitudinal axis and trajectory is the vehicle's yaw. Because the lateral scanning program is necessarily given in vehicle-fixed coordinates, the yaw angle must be zero if the correct scanning program is to be maintained relative to the trajectory. As the yaw angle increases, the width of the scanned strip under the vehicle decreases, until at a yaw angle of 90 deg the original 40-frame strip has degenerated into one a single frame wide.

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Perhaps more important than a precise control of attitude is the control of its rate of change. The resolution of the television system is much reduced by changes in attitude occurring during the scan of one frame. Moreover, attitude angular velocities may prevent the overlapping of sequential frames and make it difficult to locate the ground position under observation. Provided rates of change of attitude are kept low, an absolute attitude error of several degrees may not result in a significant reduction of vehicle utility. Further studies are necessary to specify more precisely the quantitative attitude requirements which should be placed on the vehicle.

Scope of Attitude Study

It is clear from the preceding remarks that some control of satellite attitude and its rate of change is required. This study will examine methods for sensing deviations of attitude from its desired value and for correcting such deviations. A number of logical possibilities for such systems have been considered and have been analysed in view of the attitude requirements presented qualitatively above. The methods which appear most promising have been singled out and recommended for further study and possible adoption. Other restrictions placed on these systems during their selection have been that the weight and power consumption be low, that reliability be high during a period of at least one year, and that they be capable of mechanization without out a lengthy development program.

The several purposes of the study can be put into better perspective by reference to a functional diagram (Fig. 1). It is seen from this that a complete study of satellite attitude errors must embrace not only the sensing and control (torquing) systems, but also the nature of the perturbation torques acting on the vehicle dynamics and the design of a control computer which governs the control torquing as a function of sensed attitude deviations.

General considerations on sensing and control systems and perturbation torques are presented in Chapters 3 and 7 following. Analytical details of some of the more promising systems are given in Chapters 4, 5, 8, and 9. Little can be said at this time about the physical form and dynamical properties of the ultimate satellite vehicle, as this question requires further treatment after more is known about the internal components.

However, before presenting the basis upon which certain systems are recommended, it is convenient to outline the prospects for attitude sensing and control that result from this study. The following sections summarize the systems study, the principal characteristics of the more promising systems, and the order of magnitude of expected perturbation torques. Recommendations for both analytical and instrument development studies are included there.

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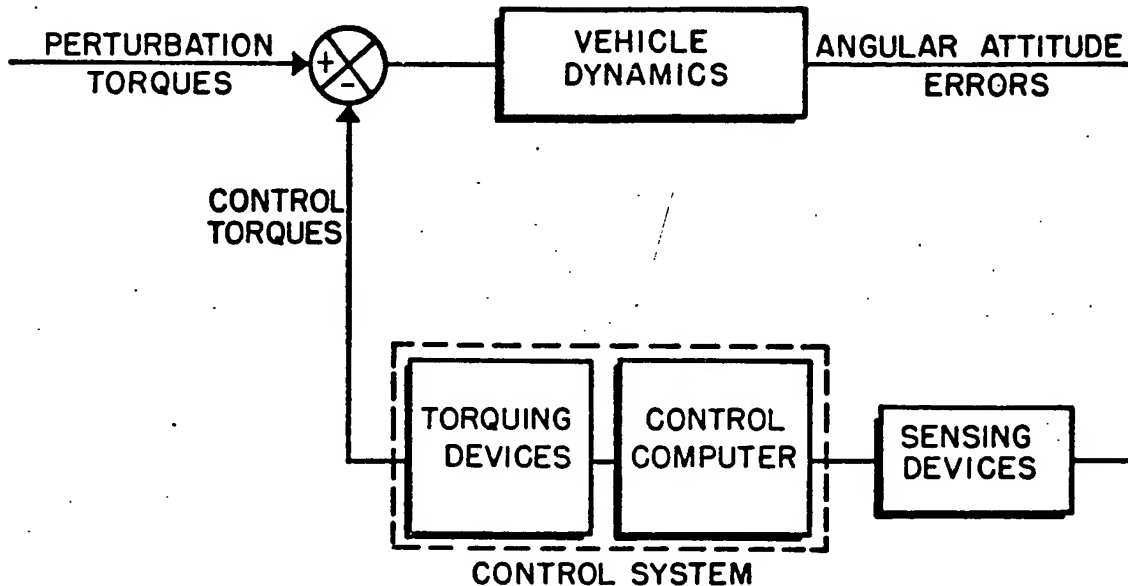
Fig. 1. Functional Diagram of Satellite Sensing and Control System

1.2. ATTITUDE PERTURBATION TORQUES

The expected attitude perturbation torques on the satellite are presented in Fig. 2. Eleven sources are listed, those with arrows beside them having a torque level well below the threshold of importance. For one of these sources, meteorite bombardment, it must be recognized that,although the average torque is small, when an impact does occur it may be expected to introduce a significant transient into the attitude deviation. The following facts stand out from this table of perturbations:

1. Aside from the reaction or gyroscopic coupling effect from rotating parts, the torque magnitudes are small, in the range, 10^{-6} to 10^{-5} newton-meter (1 newton-meter $= 10^7$ dyne-cm $= 0.738$ ft-lb). A control system designed for torques of 10^{-4} newton-meter in pitch and yaw and 10^{-5} newton-meter in roll will probably suffice, with an ample margin of safety, from this point of view alone. If pains are taken to remove or reduce perturbation torques, perhaps these requirements can be scaled down by an order of magnitude.

2. The major perturbation may be from rotating parts in the vehicle. Practically, this means that a control system cannot be designed quantitatively

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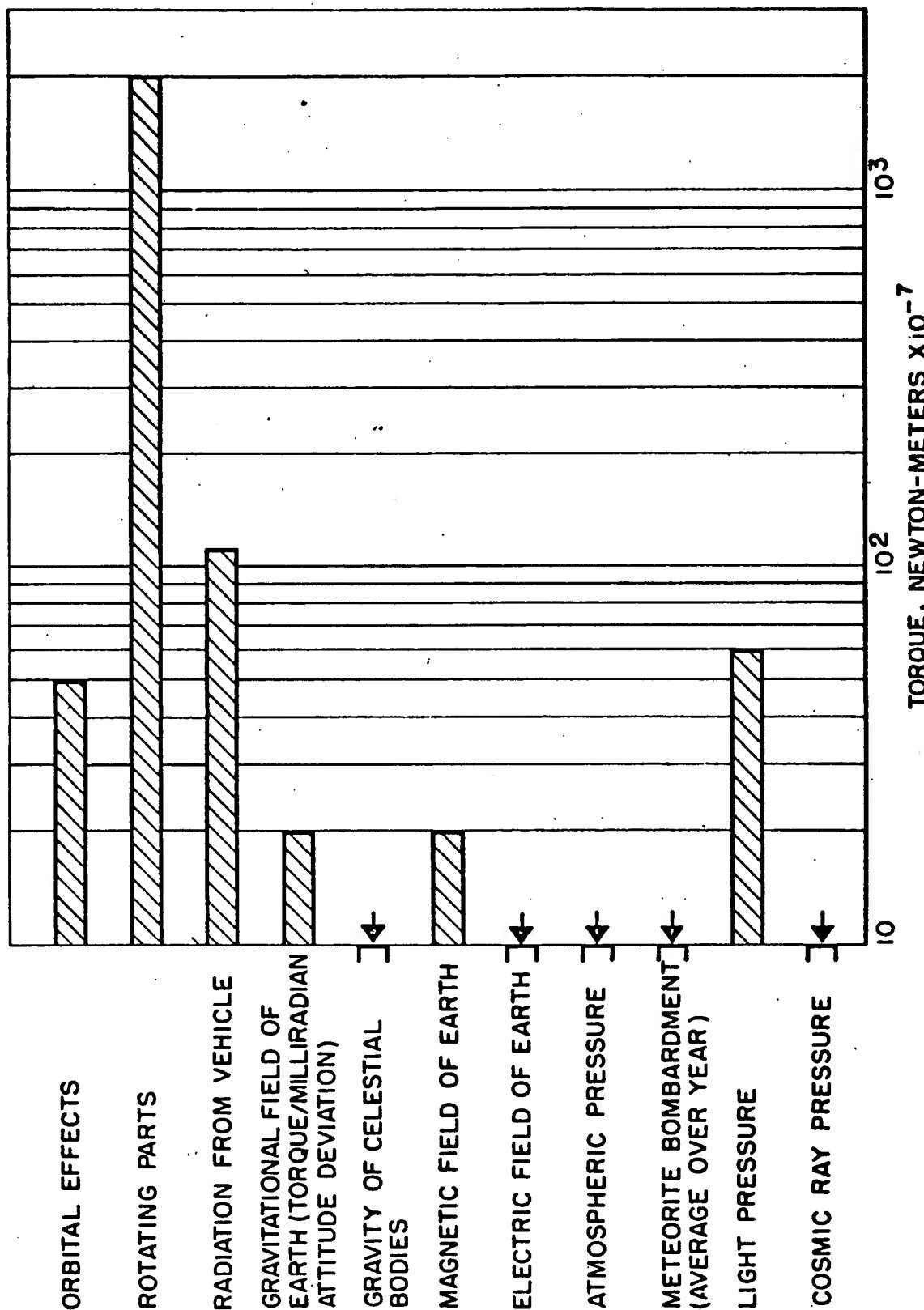
Fig. 2. Estimated Maximum Attitude Perturbation Torques

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until a much more detailed configuration design than now available is completed. The greatest care must be taken to avoid attitude excitations in any such design, as the attitude perturbation torque levels are extremely small by conventional standards.

Another fact, which is not evident from the table but which appears from the analysis of Chapter 2, is that some of the perturbation torques may be persistent in direction over long periods of time. This means that the pitch flywheel, in particular, may be inadequate to control the attitude unless it is supplemented.

1.3. ATTITUDE SENSING AND CONTROL

As a result of the work reported here, it may be said that

1. No reason has been discovered why a satisfactory system cannot be designed and constructed within the existing technological framework, without the necessity for a major research program.

2. Some possibilities have been conceived for complete sensing and control methods which are felt to be feasible in principle, reasonably simple in mechanization, and sufficiently economical in weight and power to be incorporated into the vehicle without undue difficulty.

Recommended Systems

The systems which have been studied as possibilities for attitude sensing and control are shown schematically in Fig. 3 and 4 respectively. These systems are divided from left to right, into four categories: all of those considered; those which are unsatisfactory or infeasible; those which might be used, but which are felt to require considerable further investigation or development; and those which are recommended for immediate physical study. A more detailed description of the systems recommended for sensing the vertical and yaw and of the principal and supplementary attitude control systems is presented in the following pages.

Vertical Sensing

A gyro with vertical spin axis is torqued to precess at constant orbital rate about the binormal to the orbital curve. During the satellite day, this reference is monitored by a horizon scanning system which senses the optical discontinuity between the earth's disk and ambient space. Gyro drift, orbital regression, and torque bias errors are corrected by the monitoring system. General specifications are as follows:

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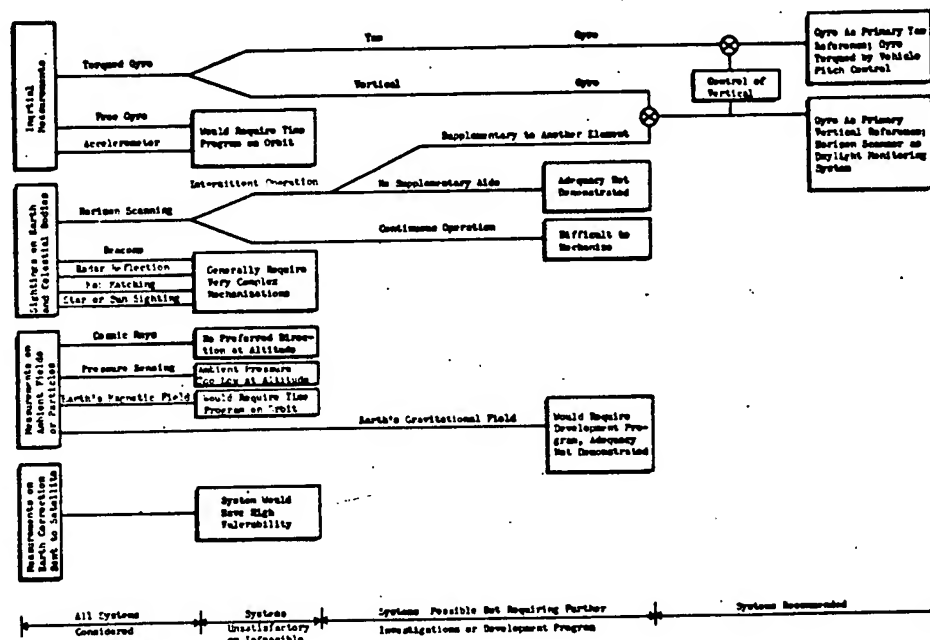
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Fig. 3. Sensing Systems for Satellite Attitude
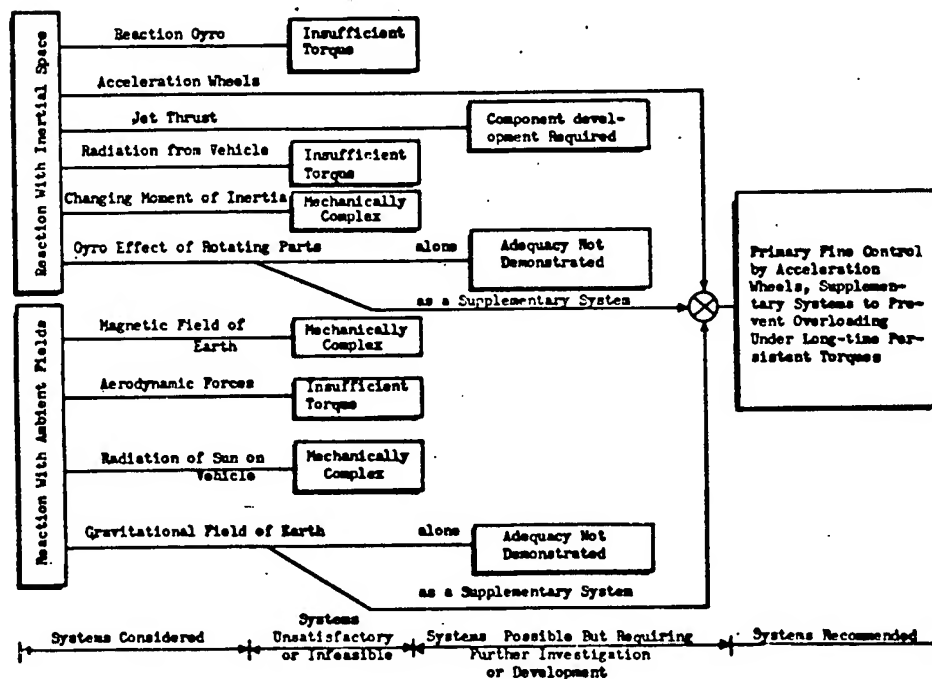
Fig. 4. Control Systems for Satellite Attitude

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1. Gyro: drift rate not to exceed 4 milliradian/hr; estimated weight including torquer less than 0.5 kg; operating power less than 5 w.

2. Horizon scanner: resolution of horizon to be within 5 milliradians; estimated weight for four units less than 5 kg; operating power less thar 30 w, using vacuum tubes, or 10 w using transistors. The resolution requirement is a tentative estimate, subject to revision.

The combined system is expected to have an RMS error less than 5 milliradians, as a relatively long smoothing time will be used.

It is believed that this system can be mechanized by existing design techniques. The required gyro life can be realized by a relatively low spin velocity and ball-bearing spin-axis bearings. Daylight operation of a horizon scanning system is known to be attainable. Long-time reliability should not be a problem if transistors are used in the scanner.

No operational limitations are imposed on the system as to altitude of operation, inclination or ellipticity of the orbit (although reconnaissance requirements restrict the latter). The system combines relative mechanization simplicity with reliability, low vulnerability, low weight and power consumption, and should integrate easily with the guidance system used for the trajectory and transition phases. Testing by conventional means (on the earth's surface or by aircraft) is possible.

Yaw Sensing

A gyro can be so constrained that it is torqued about the binormal to the orbital curve by the control torques applied to the vehicle to stabilize the vertical. With proper damping, it finally will aline its spin axis with the binormal and serve as an indicator of yaw. The drift rate of such a system should not exceed 4 milliradian /hr. With this specification on drift rate, the system is expected to have an error of no more than 1 to 2 milliradians in excess of the error in controlling the vertical. Estimated weight is about 0.25 kg and operating power less than 5 w.

It is felt that this system can be mechanized by existing design techniques. The required gyro life can be realized by a relatively low spin velocity and ball-bearing spin axis bearings. The advantages are the same as those for vertical sensing.

Principal System for Attitude Control

Three flywheels are mounted with their spin axes along the three principal axes of the vehicle. If deviations about any of these axes are sensed, the corresponding wheel is accelerated in the deviation direction relative to the vehicle frame. The reaction torque on the vehicle returns the deviation to

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zero. The maximum angular momentum storage of each wheel should be of the order of 1 kg-m^2/sec. This should be adequate unless there are significant perturbations which are persistent in direction and cannot be removed or reduced. Estimated weight for three wheels is about 25 kg. Specifications on the control computer have not been formulated.

It is felt that this system can be mechanized by existing design techniques. Careful design practices must apply to the control computer to maintain reliability and low power consumption. The control system is capable of exerting control torques through a continuous range of values including zero, thus permitting a fine, damped control of attitude oscillations. Because it operates by interaction with inertial space, it is free of the influence of any ambient fields which may change with altitude or orbit and does not require a knowledge of satellite position. Moreover, it can be tested on the surface of the earth.

Supplementary System for Attitude Control

A vehicle configuration is required which is elongated in a geocentric direction. Such a configuration is inherently stable in roll and pitch, because the differential gravitational attraction on the two ends of the vehicle provides a restoring force when the attitude is perturbed. The rotating parts within the vehicle are alined with the pitch axis, so that the roll and pitch stabilizing torque of gravity interacts with the rotating parts, providing a gyroscopic couple to aline the satellite correctly in yaw. Roll, pitch and yaw alinement couples are obtained by the configuration design alone, without need for sensing equipment or weight or power in excess of the normal operational requirements.

The question of feasibility of the stable attitude configuration from the point of view of other operational requirements of the vehicle is beyond the scope of this report. If such a system were used alone, it would be unable to damp out oscillations induced by perturbation torques. It is not certain that the maximum oscillations could be held within tolerable bounds. However, it is to be regarded as a most important, if not essential, supplement to the regular control system.

Design Recommendations

Aside from the recommendation of the use of a stable attitude configuration, the study of perturbation torques leads to the following design recommendations for the vehicle:

1. Rotating parts should be avoided unless rotation axis is alined with pitch axis, or unless there is a careful torque balance by means of a counter-rotating part.

2. Rotating loops of fluid should be subject to the same conditions as above.

3. Low impedance closed electrically conducting loops should be avoided in the vehicle.

4. The area exposed to meteorite bombardment should be minimized by the free use of open framework where other design considerations permit.

5. Heat radiating areas which are symmetric about a principal axis of the vehicle should be used.

Recommendations for Further Study

The following questions appear to deserve further and more detailed study than could be carried out in this preliminary survey. These are all in the nature of specific design studies, except the necessary supporting studies.

1. General Studies
   a. Definition of operational requirements on attitude.
   b. Definition of operational requirements on rate of change of attitude.
   c. Determination of orbital curvature and torsion as a function of the attitude, attitude rate, altitude, and heading errors at the beginning of the orbital phase.
2. Sensing System Studies
   a. Torquing system for vertical gyro.
   b. Coupling between vertical gyro and horizon scanning monitor.
   c. Error analysis for vertical sensing system.
   d. Error analysis for yaw sensing system.
   e. Establishment of performance criteria and choice of optimum design parameters.
3. Control System Studies
   a. Mechanization of control equations.
   b. Error analysis for acceleration wheel system.
   c. Establishment of performance criteria and choice of optimum design parameters.
4. Configuration Studies
   a. Optimization of design as a supplementary control system.

The following programs are indicated to insure the feasibility of the recommended systems:

1. Design and construction of a gyro to demonstrate low drift rate and long life.
2. Design and construction of daylight horizon scanner to demonstrate attainment of the resolution specified.
3. Design and construction of an acceleration wheel to demonstrate achievement of desired voltage-torque relationship and long life.

2. ATTITUDE PERTURBATION TORQUES

2.1. TORQUE SOURCES

It is apparent that some knowledge is required of the nature and size of torques which act to perturb the vehicle attitude from its desired value. Two applications may be expected from such an analysis. First, the vehicle configuration can be designed more rationally so as to reduce perturbations in every way possible by the design alone. Second, the control system to be used can be chosen and designed with assurance that it will be able to maintain the prescribed attitude.

A number of sources of perturbation have been considered:

1. Orbital curvature and torsion.
2. Rotating parts.
3. Radiation from vehicle.
4. Earth's gravitational field.
5. Gravitational fields of celestial bodies.
6. Earth's magnetic field.
7. Earth's electric field.
8. Atmospheric pressure.
9. Meteorite impact.
10. Light pressure.
11. Cosmic ray bombardment.

The estimates of torque magnitudes for some effects are necessarily crude because of the present dearth of exact information on the properties of the atmosphere at the orbital altitude. Moreover, the results generally depend upon the vehicle configuration which is assumed. As a final configuration is not known, the torque estimates are made on the basis of the tentative configuration presented in Ref. 1 or upon a subjectively "reasonable" estimate of configuration variables not specified there.

The magnitudes and natures of the perturbation torques expected from each source are treated in the remainder of the chapter.

2.2. TORQUES FROM REACTION WITH INERTIAL SPACE

Torques from Orbital Effects

The motion of the center of mass of the vehicle cannot directly affect the vehicle's orientation. In fact, the angular velocity in space is fixed in the absence of external torques, regardless of the orbit. However, the desired

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attitude is not space fixed and does depend upon the relation of the orbit to the earth. A changing orbit changes the desired attitude, so that control torques must be applied to keep the vehicle attitude properly fixed relative to the earth. The tendency of the vehicle to depart from its desired attitude appears to an observer in the satellite exactly as if it were caused by a perturbation torque. Because a plane circular orbit permits the satellite to keep the desired attitude without controlling torques, one may consider the departure of the orbit from this elementary form as giving rise to a set of perturbation torques acting on the vehicle. It should be remembered that these are actually "equivalent torques" in the sense that their negatives would have to be applied to the vehicle in order to prevent attitude deviations. In particular, these orbital perturbation torques depend upon the curvature and torsion of the orbit and upon the time rates of change of curvature and torsion. Therefore, the nature of the orbit as a space curve must be determined in order to estimate the orbital perturbation torques.

A perturbation calculation taking into account the ellipticity of the earth has been made by Brower (Ref. 2), but he limits his discussion to third order terms in orbital inclination. Moreover, his results are not in a form which can be used readily to obtain torques. Further studies of these perturbation torques are required to find the torsion and curvature of typical orbits. Tentative estimates of torque from this source, based on Brower's results, lie in the range 1 to 5 x 10^{-6} newton-meter (10 to 50 dyne-cm).

Reaction Torques from Rotating Parts

Rotating parts within the vehicle can cause perturbation torques either by their acceleration or by their gyroscopic interaction with the fairly uniform pitch angular velocity. Magnitudes cannot be given in the absence of detailed knowledge of configuration, but such torques can be a significant source of attitude perturbation. This is best illustrated by some characteristic examples.

In the search phase, the satellite antenna is supposed to rotate about the vehicle zenith axis at 3 rps. If its moment of inertia is 10^{-2} kg-m^2 and the vehicle pitch velocity is 10^{-3} radian/sec (100-min orbital period), then a roll torque arises from the gyroscopic coupling between these rotations, its magnitude being of the order of 10^{-4} newton-meter (1000 dyne-cm).

Suppose that a certain amount of fluid circulates within the vehicle, traveling in a loop with an equivalent radius of 1 m, that its total mass is 1 kg, and that its path velocity along the tube is 0.1 m/sec. Then the gyroscopic interaction torque analogous to that from the rotating antenna is of the order of 10^{-4} newton-meter (1000 dyne-cm).

These examples suffice to show that under reasonable assumptions the perturbation torques of this class may be large. They may be reduced or removed by the addition of counterrotating parts but only at the expense of the addition of mass. Alternately, some may be reduced by alining the axis of rotation with the vehicle pitch axis. In general, however, all that presently can be said about such torques is that they can, and must, be reduced by careful configuration design. Because it is not known how low a level can be attained, the torque estimates of Chapter 1 include a large factor of safety.

Torques Resulting from Radiation from the Vehicle

There are three principal sources of energy radiation from the vehicle: heat dissipation involved in the vapor cycle of the orbital power plant, dissipation of heat which was previously absorbed from the sun, and microwave communication with the earth.

According to Ref. 1, pp. 59 and 125, the rate of heat dissipation through the skin necessary to operate a 500-w engine must be about 5.87 kw (20,000 Btu/hr). The radiant energy for communication should be less than 10 percent of this, and a vehicle skin of fairly high reflectivity keeps the absorbed energy from the sun to a few percent of the heat engine dissipation energy. For illustrative purposes, suppose that the total radiation is about 6 kw, which corresponds to a thrust of 6×10^3 newton-meter /sec $\div 3 \times 10^8$ m/sec $= 2 \times 10^{-5}$ newton (2 dynes). If the centroid of the radiation area is located at a distance of the order of 0.5 m from the projection of the vehicle center of mass on the radiation area, then the reaction torque on the vehicle from this radiation is of the order of 10^{-5} newton-meter (100 dyne-cm).

Because the plane of the vehicle's longitudinal axis and its zenith direction is likely to be a plane of symmetry, the center of mass and the centroid of radiation area are likely to lie on or near the longitudinal axis. Therefore, this perturbation torque is one of pitch, and is persistent in direction.

This torque can be reduced by careful design of the radiating surface without major changes in the tentatively proposed vehicle configuration. If desired, however, it could even be augmented to compensate for any other persistent pitch torques on the vehicle.

2.3 TORQUES FROM REACTION WITH AMBIENT FIELDS AND PARTICLES

Torques Resulting from the Earth's Gravitational Field

It is easy to show that the stable attitude of the vehicle in its orbit is one in which the mass distribution of the vehicle lies generally above and below the center of mass. This is because the mass elements below the center of mass

are acted upon more strongly by gravitational than by centrifugal forces (which are just balanced at the center of mass). The converse is true for elements above the center of mass. Thus, any perturbation of such an attitude results in a restoring torque.

On the other hand, the conventional configuration in which the longitudinal axis is along the trajectory is one of unstable equilibrium and any small attitude deviation will grow.

The effective "spring constant" of the earth, i. e., the unbalance torque per unit angular rotation experienced for small attitude errors, can be calculated only if a configuration is known. However, one can make a useful estimate by considering the vehicle to be composed of a pair of equal point masses lying (in the equilibrium case) along the trajectory. Then, if these masses have earth weight W_e and distance apart a, and if the line joining them makes an angle θ_2 with the trajectory, the unbalance torque is

$$ L = \frac{3a^2 R_o^2}{8R^3} W_e \sin 2\theta_2 \tag{1} $$

Here R_o is the earth radius and R the orbital radius. For the case of a 560-km orbit, W_e = 500 kg, a = 1.5 m, the unbalance spring torque is 2×10^{-3} newton-meter/radian (20 dyne-cm/milliradian).

This unbalance effect could be removed or reversed, making the desired attitude stable, by a vehicle configuration in which the lumped masses representing the vehicle lie above and below the center of mass, rather than fore and aft of the center of mass. This possibility is developed further in Chapter 7 as a means of control.

Torques Resulting from Gravitational Fields of Celestial Bodies

There are two ways in which such gravitational fields may affect attitude. They may affect it because of their differential attractions on the various portions of the vehicle or because they modify the vehicle's orbit. Insofar as they affect the orbit, they may affect attitude indirectly as explained previously.

A study by L. Spitzer (Ref. 3) has shown that the orbital perturbations caused by the moon and sun are overshadowed by perturbations resulting from the oblateness of the earth. Therefore, one infers that the equivalent perturbation torques of these orbit changes are small compared to those already considered. One also infers that the differential gravitational effect from the sun, moon, and other celestial bodies is negligible compared with that from the earth, which has been analysed above.

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Torques Resulting from the Earth's Magnetic Field

The earth's magnetic field will interact with the satellite through the permanent magnetic moments of internal components and with any fields arising from the operation of electrical equipment. Additional torques arise from the eddy currents induced in the various conductors which comprise the vehicle.

All of these effects are difficult to appraise before a complete configuration is given. Quantitative estimates of the internal fields do not seem to be available, and their calculation would require the prior measurements of typical effective magnetic moments, both permanent and those arising from the stray fields of motors and generators. The torque arising from the susceptibilities of components (i.e., from the tendency of the earth's lines of flux to take paths of lowest reluctance through the components) is equally difficult to estimate.

A little more can be said about the possible effects of eddy currents induced in the vehicle by the earth's field. As seen from the satellite, the earth's magnetic field vector describes a cone with its axis off the port beam of the vehicle and semiangle equal to the orbital inclination relative to the magnetic equator. The period of rotation of this field vector about the cone axis is equal to the orbital period. The vehicle will experience a retarding force along its negative path direction, and because of the constant field component along the pitch axis, a constant positive pitch torque. In general, roll and yaw components of torque will also be present, but they will be periodic and smaller than the constant pitch torque. Moreover, it can be shown that the retardation force is negligible over the period of one year. Therefore, only the constant pitch torque need be considered as a possibly significant perturbation.

Eddy current calculations are all but impossible for bodies of any but the simplest shape. Therefore, to estimate the torque magnitude, the following assumptions are made:

1. The vehicle is in a field of constant magnitude, which (as seen from the vehicle) is rotating around it with a uniform angular velocity.
2. The vehicle may be replaced by an equivalent spherical shell of properly chosen radius and resistivity.

With these assumptions, a formula given in Ref. 4 may be used to find the pitch torque. The computed torque depends critically on the assumed constants of the equivalent sphere. Estimates of this torque have been made in the range 10^{-4} to 2×10^{-6} newton-meter (0.1 to 20 dyne-cm) for a polar orbit, depending upon the intuitive feelings of the investigators as to the kind of sphere to which the vehicle might be equivalent. One sees, therefore, that this source of perturbation probably is not of significance.

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Fortunately, it is fairly simple to reduce eddy current effects to well below the threshold of importance. One need merely break the vehicle shell by one or more insulated strips in order to destroy any large possible paths for current loops. Similarly, the effects of stray fields from electrical components can be reduced by careful design.

Torques Resulting from the Earth's Electric Field

The potential gradient of the earth's field has been measured at various points on the earth's surface and on the lower regions of the space surrounding the earth (Ref. 5). Surface measurements vary from something less than 100 v/m to about 300 v/m. In addition to these area variations, there are daily and seasonal variations at any given point.

Measurements indicate that the gradient is decreasing with altitude at a much greater rate than would be predicted if the field were that of a charged sphere. This is explained on the basis that the conductivity of the atmosphere increases with altitude. In the region of the ionosphere and above, the atmosphere is considered to be a perfect conductor, so that no potential gradient could exist in this region. However, ion clouds resulting from particle bombardment may exist at altitude. There is no general agreement as to whether a body would become charged on flying through such clouds, or if so, what its polarity might be.

On the basis of the previous discussion, it would seem that the satellite vehicle would not at any time find itself in a region of a potential gradient. However, it would be of interest to know what the effect on the vehicle would be if a gradient did exist. Apparatus for measuring the potential at any point depends upon some sort of "collector" device, such as a sharp metallic point. It is conceivable that such collectors could exist on the vehicle, so that if the vehicle were to be traveling through a more or less constant gradient, a potential difference would exist between points on the vehicle. Hence, a current would flow. The effect would be similar to that of a finite length of a current-carrying wire traveling through a magnetic field. Because a closed current loop does not exist in the vehicle, there would be no torque exerted with respect to vehicle axes, but rather a force acting at the center of mass producing a translatory motion. Therefore, electric field effects should not be considered as a source of attitude perturbation torques.

Another, more subtle, effect might exist in the electric or magnetic polarization of the vehicle and the resultant interaction with the earth's magnetic or electric field. However, it does not seem feasible or necessary to try to analyze this effect.

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Torques Resulting from Atmospheric Pressure Field

Despite the tenuous atmosphere at the expected orbital altitude (560-800 km), the satellite will encounter a certain air resistance which may be characterized as that due to atmospheric pressure. In a normal attitude with an axis of symmetry along the trajectory, the effect of this resistance will be purely to decrease the satellite's path speed and will not give rise to any attitude changing torques. The situation is analogous to that discussed in connection with the effect of the earth's gravitational field. The desired attitude is one of equilibrium, but the equilibrium is unstable. If an attitude perturbation from the desired value takes place, a torque will begin to act which tends to increase it. Thus, one calculates an effective spring constant for this case, rather than an absolute perturbation torque.

There seems to be no simple way to calculate such a quantity for travel through an atmosphere of such low density that each molecular contact involves a collision problem. However, if one uses classical aerodynamic theory in a conventional kind of calculation, he may hope that the ensuing estimate of spring constant is not worthless.

Consider the moment per unit yaw angle acting on an ogive, neglecting any boattail effect. According to the formula of Max Munk (Ref. 6 and 7), it is the product of volume, air density, and path speed. For typical satellite conditions, using the configuration of Ref. 1 and a density estimate deduced from the data of Ref. 8, one may say that

$$\text{Volume} = 6 \text{ cu m}$$

$$\text{Air Density} = 0.5 \times 10^{-15} \text{ kg/cu m}$$

$$\text{Path Speed} = 0.76 \times 10^{4} \text{ m/sec}$$

Then the spring constant is 2×10^{-7} newton-meter/radian (0.002 dyne-cm/milliradian), which is negligible compared to the spring constant of the earth's gravitational field.

Torques Resulting from Meteorite Bombardment

As the satellite moves along its orbit, it encounters three classes of meteorites:

1. Those of dust size, which probably have no average preferential direction of motion and influence the satellite in exactly the same way as the molecules of the ambient atmosphere.

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2. Those of medium size, which produce a significant impulsive torque on impact but do not penetrate the vehicle.

3. Those of large size, which penetrate, and probably destroy, the vehicle.

Encounters of the last class are rare, but they are of obvious importance. However, upon such an encounter, it is unlikely that the functioning of the attitude control system will be of further interest. Only the first two classes of encounters are considered here.

Encounters with meteorite dust may be analysed by finding the average pressure from the discrete impacts of the dust and comparing it with the pressure of the atmosphere. Consider a unit surface with its normal directed along the normal to the satellite orbit. A meteorite which passes through this surface will transport across it an average component of momentum in the normal direction equal to the product of meteorite mass and its rms velocity component across the surface. This mean square component is about a third of the mean square velocity. Assuming a velocity of 150,000 fps (Ref. 9), and neglecting the vehicle velocity in order to obtain an answer independent of direction (the meteorite data of Ref. 9 are open to serious question, and are used here merely because more reliable information seems not to exist), the rms velocity across the surface is found to be 88,000 fps. The total momentum transfer across a unit area per unit time is the sum of the momenta from meteorites of each mass class, weighted by the number of the class expected to strike a unit area in unit time. This rate of momentum transfer is just the pressure on the area.

It can be shown from the data of Ref. 9 that the pressure from meteorites of each visual magnitude in the range 0 to 30 is about 6×10^{-14} newton/sq m (6×10^{-13} dyne /sq cm). Meteorites of other classes than those considered will be encountered only with very low probability during the course of a year. This compares with an atmospheric pressure of the order of 10^{-6} newton/sq m, and indicates that the effects of meteorite dust may be neglected.

This conclusion is not changed if one accepts new estimates of the amount of meteorite dust present, which are about three orders of magnitude greater than the earlier estimates (Ref. 10). Even with these, the dust density would correspond to a pressure of the order of only 10^{-10} newton/sq m, as compared with 10^{-6} newton/sq m for the atmosphere at altitude.

The problem of encounter with larger meteorites can be investigated on the basis of Ref. 10 for average meteoric conditions. Using the method of calculation outlined there, considering no meteorites of mass less than 10^{-8} kg, one can obtain (Fig. 5) a probability $P_1(m)$ of at least one hit per year on an area of 1.1 sq m by fragments of mass m or greater. The choice of 1.1 sq m (12 sq ft) for the area on which hits are made is rather arbitrary, but the results can be converted to other areas.

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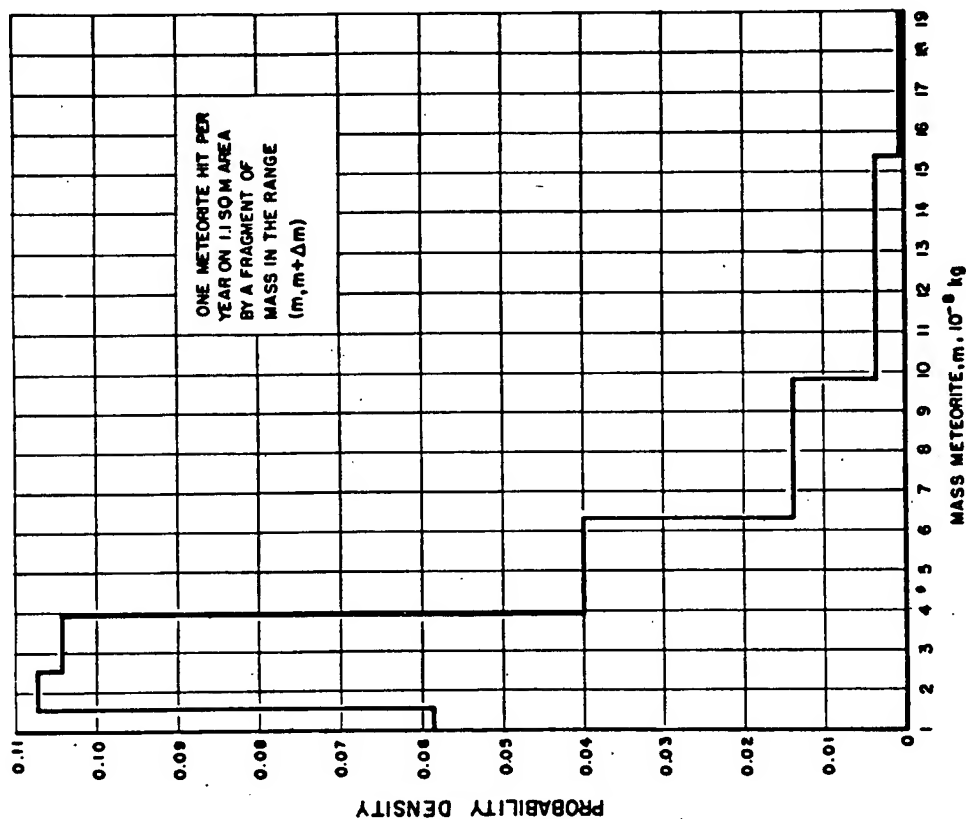
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Fig. 6. Probability Density for Meteorite Hits
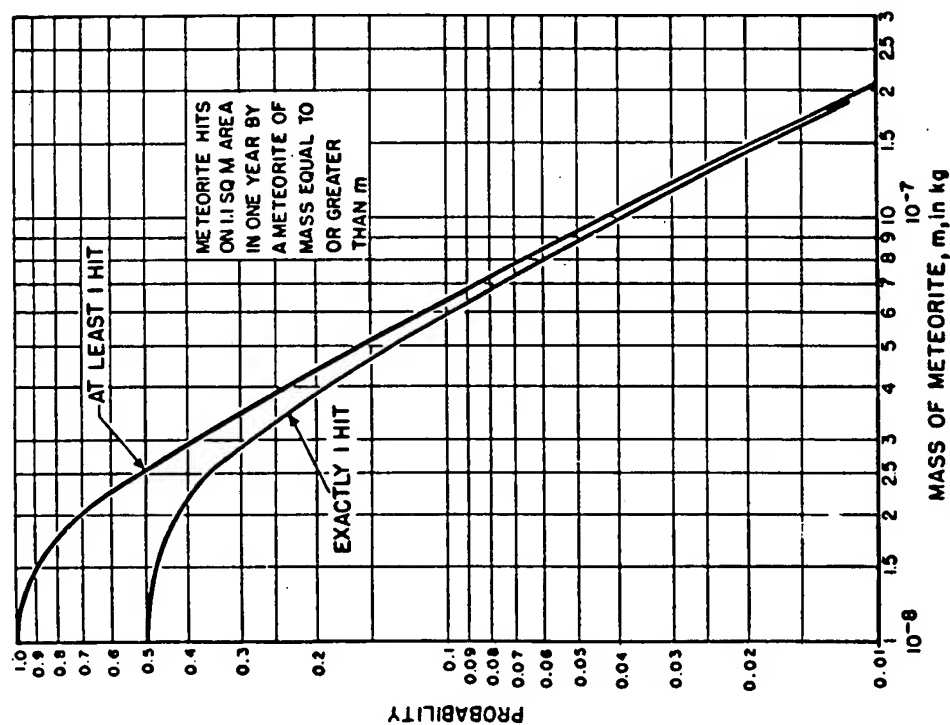
Fig. 5. Probability for Meteorite Hits

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If it is assumed that hits on the area are independent, then the probability of exactly one hit per year on the area by fragments of mass m or greater is just $P_o/(1 + P_o)$. The corresponding probability density $p(m)$ is given approximately, as a histogram, in Fig. 6. Masses are related to visual magnitudes (which are the independent variables for most meteor data) under the assumption that the meteorites in question are stony, with a specific gravity 3.4, because the corresponding information does not seem to be available for other classes of meteorites.

Finally, one may suppose that the average distance of impacts from the center of mass corresponding to a 1 sq m area is about 0.35 m. Also, assuming that all meteorites have the common velocity 4.6×10^4 m/sec (125,000 fps),

the expected impulse is $\int_0^\infty m(4.6)(0.35)(10)^4 p(m) dm$.

An approximate numerical integration based on Fig. 6 gives an expected impulse per year on 1.1 sq m equal to 3×10^{-4} newton-meter-sec (3,000 dyne-cm-sec), which should be correct in order of magnitude. For a larger area, say 10 sq m, and a correspondingly greater value of mean striking distance, say 0.7 m, the corresponding expected impulse is of the order of 6×10^{-3} newton-meter-sec (6×10^4 dyne-cm-sec).

An impulse of this size is significant, but it occurs only once a year on the average and may represent only a transient disturbance of the vehicle.

Torques Resulting from Radiation from the Sun

Radiation from the sun, the principal source of radiation falling on the vehicle, falls on the outer limits of the earth's atmosphere at the rate of 1.93 cal/(sq cm)(min) or 0.13 joule/(sq cm)(sec)(Ref. 11). This is essentially the total radiation power on the satellite. If the coefficient of reflection of the vehicle is unity, the radiation pressure is, therefore, 2 x 0.13 joule/(sq. cm) (sec) $\div c \doteq 8 \times 10^{-6}$ newton/sq m. A relatively high coefficient is probably required in order to prevent the satellite interior from overheating. The radiation which is absorbed and reradiated from the cooling coils on the earth side does not contribute a significant net torque on the average, as mentioned previously.

It is worth while to analyse the geometry of this situation in some detail, in order to bring out the persistent nature of this torque over relatively long times.

As viewed from the satellite, the sun will appear to follow a right cross section of a circular conical surface of semivertex angle α_o and axis along one beam of the vehicle (Fig. 7). The sun will be obscured by the earth while
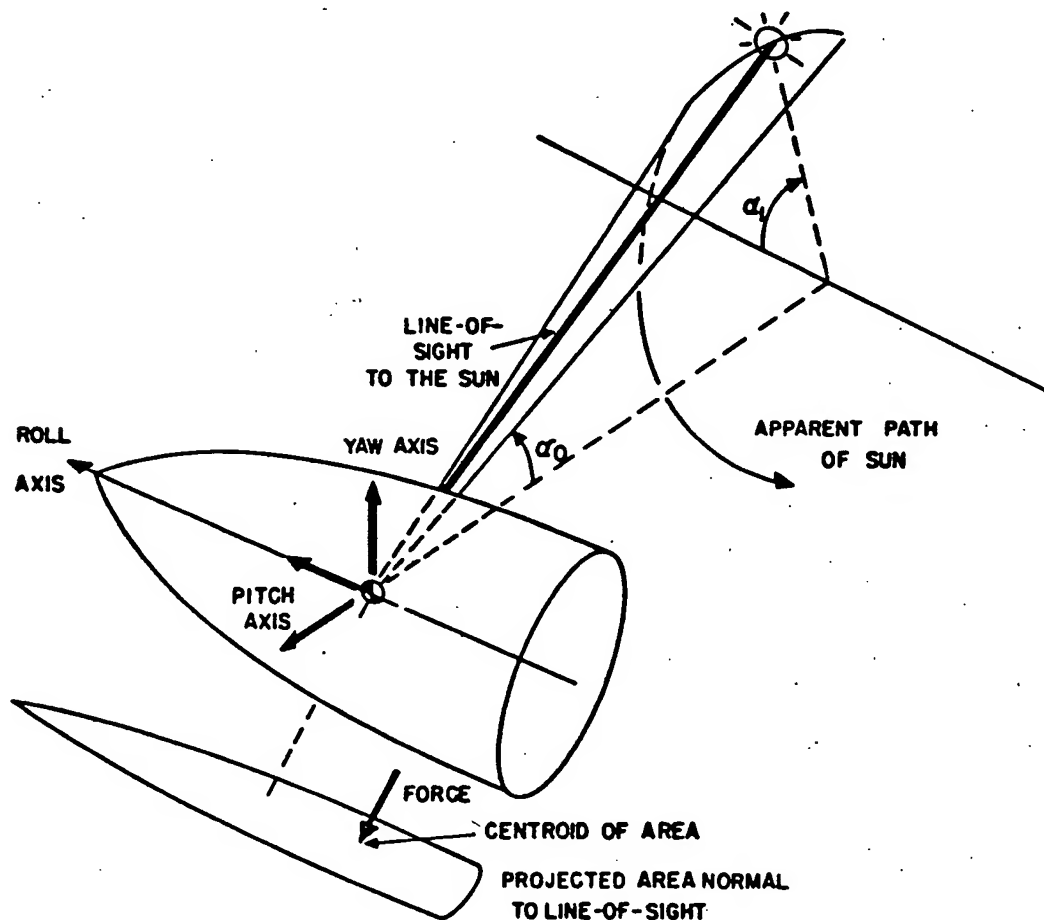
Fig. 7. Cone of Sun and Projected Area of Satellite

it is in the lower portion of this circular path, although it will be visible for some angle below the horizontal plane because of the dip of the horizon. Therefore, light pressure will begin to act on the vehicle each satellite "morning," coming in along a line approximately α_0 away from the bow. The line-of-sight to the sun moves up and toward the stern during the satellite "day," passing the transverse section of the vehicle at an altitude inclination α_0 and setting at an angle approximately α_0 from the stern. The whole cycle of the satellite day (i.e., sunrise to sunrise) takes place in one orbital period of about 100 min. The angle α_0 slowly changes through all possible values in the range $[0, 2\pi]$ as the orbital regression turns the "plane" of the orbit. One may speak of a satellite "year" as the length of time taken for this α_0-cycle. According to Ref. 1, this is about eighty days for an orbital altitude of 560 km and an orbital inclination of $\pi/3$ radians.

The light pressure may be considered to act uniformly over the maximum area of the vehicle projected onto a plane normal to the instantaneous line-of-sight to the sun. Thus, it may be considered to act at the centroid of this area with a force equal to the pressure times the projected area. The perturbation torque depends on the distance from the vehicle center of mass to the area centroid. Pitch and yaw components of torque exist whose amplitudes depend upon the angles α_0 and α_1. The ratio of pitch to yaw torque is $\sin \alpha_1 \tan \alpha_0$, so that it changes during the day from zero to a maximum of $\tan \alpha_0$ and back to zero. In general, these torques act in the same direction all day and in this same direction over a period of one half the regression period (the time it takes $\sin \alpha_0$ and $\cos \alpha_0$ to change sign). Thus, they appear to the control system over periods of several days almost as persistent torques rather than as periodic torques, and they may cause the capacity of the control system to be exceeded if they have sufficiently large magnitudes.

The torque magnitudes cannot be calculated without a knowledge of the exact vehicle configuration. However, a sample calculation gives some notion of the magnitudes. Assume, for example, that the projected area is 3 sq m, and the distance from the center of mass to the centroid of projected area is 0.5 m for $\alpha_1 = \pi/2$. Then the perturbation torque is 6×10^{-6} newton-meter (60 dyne-cm). The pitch and yaw components, of course, depend upon the value of α_0 existing on the particular day. Twice during the year, each component will appear as this maximum torque. It is likely, moreover, that this is near the maximum torque magnitude to be expected during any day, so that it represents an upper bound on perturbation torques due to incident radiation. It is unlikely that even a nonqualitative change in configuration can change it by a factor of ten. On the other hand, this perturbation torque can be removed entirely by changing the configuration so as to have appropriate geometric symmetries about the center of mass, e.g., to a sphere or cylinder.

Torques Resulting from Cosmic Ray Bombardment

Upper atmosphere data currently available (Ref. 12) indicate that the energy of cosmic ray bombardment at the satellite will be of the order of 2 bev/(sq cm)(sec). As 1 ev = 1.59×10^{-12} erg = 1.59×10^{-19} joule, 1 bev $\simeq 10^{-10}$ joule. It is apparent by comparison with previous results that even if the cosmic rays are completely reflected, their bombardment effect will be insignificant compared to that of the light rays from the sun.

3. METHODS OF SENSING

3.1. CLASSIFICATION OF METHODS

A number of logically possible attitude sensing methods can be con-
ceived. In examining these methods, it is convenient to divide them into
mutually exclusive and exhaustive classes. On such division, based upon
the method of observation, follows:

1. Observations made entirely within the vehicle structure.
2. Observations made from the vehicle upon ambient field variables.
3. Observations made from the vehicle upon the earth or celestial
bodies.
4. Observations made from the earth upon the vehicle or upon signals
from the vehicle.

In effect, the first class is the class of inertial measurements. These
methods depend upon the inertial properties of material bodies: their re-
tention of a fixed direction of motion relative to an inertial frame of reference
in the absence of imposed forces. Some physical devices exemplifying this
tendency are

1. A torque-free gyroscope whose spin axis remains space-fixed.
2. A vibrating string or pendulum whose plane of vibration remains
space-fixed.
3. A proof mass which remains fixed relative to the vehicle when the
latter is in uniform motion.

As a class, systems embodying these methods suffer from the disadvantages
that they fail to distinguish between true changes in vehicle attitude and
changes in the indicated direction caused by spurious random excitations
(from which they can never be isolated completely). However, if the appli-
cation is suitably restricted and the method properly mechanized, this drift
of the reference often can be made inconsequential.

The second class embraces all methods which depend on observations
of gravitational, electromagnetic or atmospheric pressure fields at the ve-
hicle. These may have rather limited applicability. For example, the
feasibility of differential pressure sensing depends critically on the expected
altitude of satellite operation, so that this method is appropriate only to a
restricted class of satellite vehicles. The gravitational force at the satel-
lite center of mass is precisely balanced by centrifugal force, and the only
non-zero effective gravitational field at the vehicle is caused by the differ-
ential radial distances of the various vehicle elements from the instantaneous

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center of gravitational force. Thus, only observations of small gravitational field differences are possible, and the instrumentation problem can be severe.

The possibility in the third class which has the most obvious promise is that of earth-sighting, as the earth is the only body constantly in view of the satellite. However, any celestial body whose line-of-sight from the vehicle bears a simple relation to the desired satellite attitude offers a prospective reference.

It is unlikely that there are practicable methods of the fourth class which depend only on the passive observation of the vehicle by an earth station. Position determination from the ground, in conjunction with other methods, may be used: e.g., in telling the vehicle when to make certain kinds of measurements. However, observation is possible either on the regular pictures televised during reconnaissance or on special signals emitted with directional characteristics by the vehicle. Methods of the fourth class have the generally undesirable feature that they require the transmission of attitude correction information from the vehicle to the ground. This opens them to the possibility of enemy countermeasures that could destroy the utility of the vehicle.

As criteria in the choice of sensing methods, it should be noted that the following procedures are to be avoided in general:

1. Use of a time standard, unless supplemented by a correction system.
2. Methods which require a close knowledge of the orbit prior to launching.
3. Communication of attitude information between ground and satellite.

Other criteria on the physical system used for mechanization have been mentioned in section 1.1.

3.2. INERTIAL METHODS

Satellite in Free Motion

Suppose that the satellite is a free body in space, moving under the forces of certain gravitational (and possibly electromagnetic) fields. The neighborhood of its center of mass is a force-free region and is apparently acceleration-free, because the total applied force is balanced by the d'Alembert "force" on the vehicle. However, as there is no such balance of applied torques by the inertial reaction of the angular accelerations about the center of mass, it may be possible to measure these accelerations by inertial means. In principle, at least, a knowledge of these accelerations permits a determination of the satellite attitude.

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Two sensing systems of this kind are shown in Fig. 8. The angular displacement is found by integrating the angular acceleration or by the deflection of a space-fixed reference gyro relative to the vehicle frame. It is compared with the angular displacement desired at that instant or position, and an attitude error is determined. The difficulties with such systems are manifold:

1. A time program of attitude is impossible with the present lack of knowledge of attitude and orbital perturbations.
2. A position program of attitude requires position measurement on the ground, with information sent to the vehicle, or a position measuring system in the vehicle possibly surpassing other attitude sensing methods in complexity.
3. Errors in the time scale, however minute, can be expected to cause large errors in the integration during the period of a year, so that the method would have to be supplemented by a periodic attitude check using an independent method.
4. Gyro drift makes a similar independent attitude check necessary.

It is unlikely that simple inertial sensing methods for a satellite in free motion can be constructed. Satisfactory methods of this kind would have to be mechanized by systems of considerable complexity.
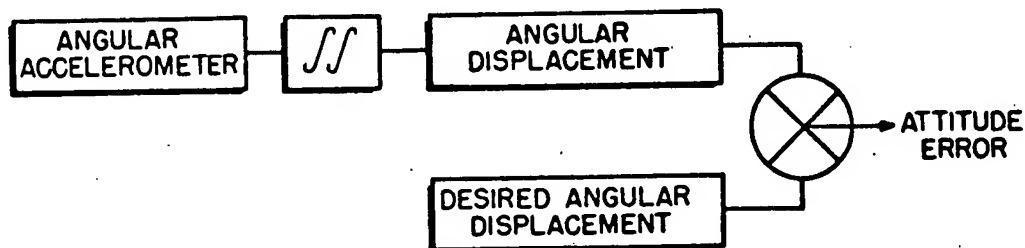
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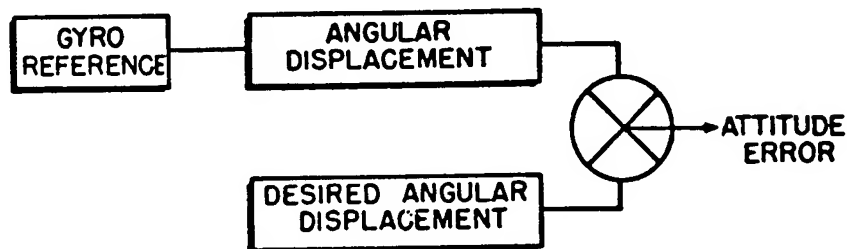
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Fig. 8. Inertial Sensing Systems for Free Vehicle

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Satellite with Partly Controlled Attitude

Prospects for inertial methods are considerably brighter when the control of one or more of the attitudes has already been established. In particular, if a stable vertical can be obtained by other means, it is feasible to monitor the remaining attitude (yaw) in this way. If a vertical is known, the situation is similar to that of the ordinary nautical navigational gyro-compass, in which the vertical is established by gravity. In the case of the gyro-compass, the force of gravity is used to torque the gyro in such a way that it alines itself in the plane of the local meridian. If the vessel were steaming along the equator, the gyro axis would be normal to the vessel's path and would indicate its yaw away from the equator. An exact analog exists for the satellite. Its orbit plays the role of an equator, while the gyro can be so arranged that it is torqued into a position normal to the orbit by the control torques applied to the vehicle to maintain the vertical (which is supposed to have been sensed by other means). Therefore, the angular displacement between the gyro axis and the longitudinal axis of the vehicle is precisely the yaw of the vehicle away from its orbital path. Chapter 4 is devoted to a more complete description and analysis of a torqued-gyro yaw sensing method.

The possibility can be considered of sensing roll deviations by the same method. It can be argued heuristically that such a system would work because in the case of both yaw and roll gyro the angular pitch motion of the vehicle applies torques to the gyro which tend to line it up with the pitch angular velocity. As in the case of the yaw gyro, the principal questions relate to (1) the ability of the gyro to detect errors caused by orbital torsion and changes in curvature, and to (2) the errors introduced by imperfect pitch control. If both roll and yaw gyros were to be used together, the additional question would arise as to their coupling by the vehicle dynamics and the stability of the over-all system. It is at this point that difficulty arises. Although the roll gyro would work in the presence of yaw and pitch control, a combined system attempting to use both yaw and roll gyros simultaneously could not maintain attitude control. In effect, there could be a drift of roll and yaw reference axes without an indication of errors in these attitudes. The pitch control torque would no longer be applied even approximately about the orbital binormal axis, and this would torque the gyros even farther away from alinement with the binormal.

Because yaw sensing by gyroscopic means seems to be necessary, a further study of the roll gyro is not recommended at present.

The remaining possibility is that of using a gyro to maintain a stable vertical. If a vertical gyro is torqued at a constant rate about the roll axis, it will precess at a constant angular velocity about the normal to the plane of the orbit. That is, it will maintain its axis alined with a ray from the center of the earth. (Orbital torsion would change this simple picture slightly, making it necessary to apply a net torque about the pitch axis as well.) If the

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exact form of the orbit were known and there were no drift of the gyro, this would provide a vertical reference at all times. However, neither the precise orbit nor the gyro drift is known in advance, and the required torque levels cannot be preset.

However, if it is possible to monitor the vertical periodically, the torque levels can be chosen to maintain the gyro vertical during the monitored periods and can be kept at the same values during the unmonitored periods. In this way, the gyro can be kept closely to its desired alinement at all times. If a system were available to monitor the vertical at all times, it could be the primary sensing system and there would be no need for the gyro reference. It will appear as a result of this study that there seems to be no method for the continuous sensing of the vertical which can be mechanized without considerable difficulty. Of those which can be mechanized easily for intermittent operation, the method of horizon scanning seems best. Therefore, a promising possibility for vertical sensing is one in which horizon scanner and vertical gyro supplement each other. This method is discussed in Chapter 6 in connection with horizon scanning.

3.3. AMBIENT FIELD METHODS

Differential Pressure

A method of detecting the departure of the vehicle's longitudinal axis from its trajectory is described in Ref. 1, p. 111. It makes use of the measurement of the dynamic pressure in an ionization gauge chamber (open to the ambient atmosphere through a short tube) and of the dependence of this pressure on the angular variation of the tube from the vehicle trajectory. The trajectory and the direction of motion of incident atmospheric molecules or ions are supposed to coincide. It is shown there that in a 560-km orbit, using the least favorable atmospheric model, the minimum angular deviation which can be detected is about 0.13 deg. The appearance of more recent information on the upper atmosphere (Ref. 8) makes it advisable to re-evaluate this pressure sensing method.

Suppose that one uses a differential pressure measurement between ionization gauge chambers an angle $\pi/2$ apart (Fig. 9). The pressures in the two gauges are respectively $CP_a \cos(\theta_1 + \pi/4)$ and $CP_a \cos(\theta_1 - \pi/4)$, where C is the amplification factor of the device and P_a is ambient pressure at the orbital altitude. The pressure difference between chambers is

$$|\Delta P| = \sqrt{2}\, CP_a \sin \theta_1 \tag{2}$$

Hence, if the angle θ_1 changes by $\delta\theta_1$, the corresponding change in differential pressure $\delta|\Delta P|$ is related to $\delta\theta_1$ by
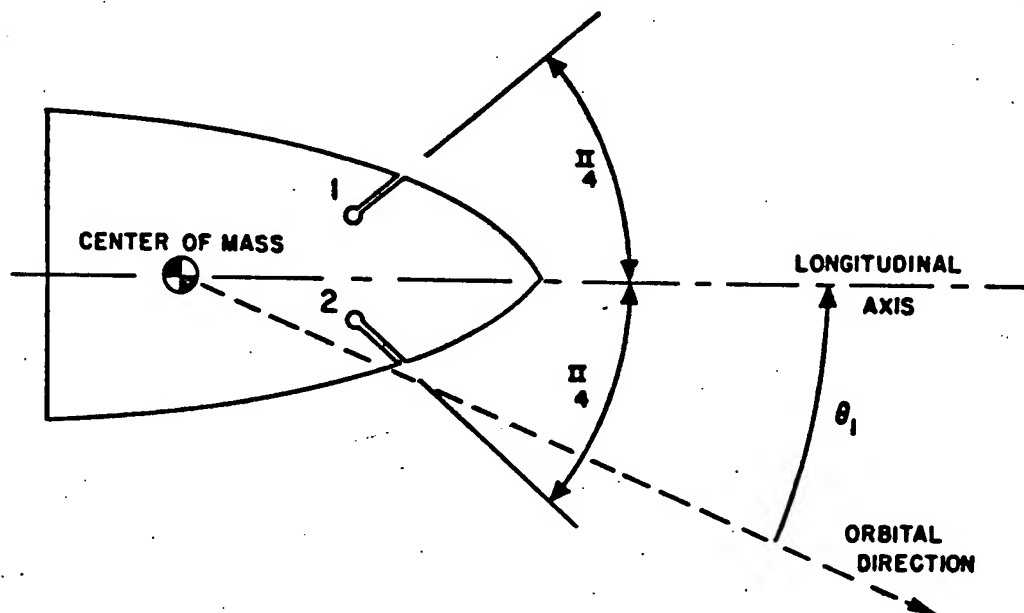
Fig. 9.   Ionization Gauge Chambers in Satellite
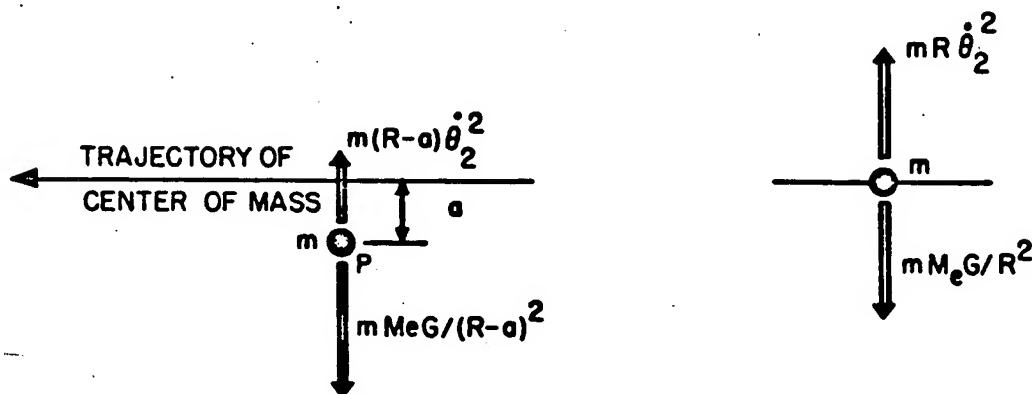
Fig. 10.   Gravitational and Centrifugal Forces on a Proof Mass in Satellite

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$$\delta\theta_1 = \delta|\Delta P|/\sqrt{2}\, CP_a \cos\theta_1 \qquad (3)$$

For a numerical example, one may take $C = 350$, $\theta_1 = 0$, and the threshold of pressure measurements as 8×10^{-8} mm of mercury. The ambient pressure assumed in Ref. 9 is $P_a = 1.4 \times 10^{-7}$ mm, whence the minimum detectable angular change, $\delta\theta_1$, is about 10^{-3} radian. However, subsequent estimates of ambient pressure at 560 km are two to three orders of magnitude lower. With the conservative (high) estimate $P_a = 10^{-9}$ mm, one obtains a minimum detectable angular variation of $\delta\theta_1 = 0.16$ radian. This is too large for most attitude sensing requirements, especially in view of the fact that attitude control is not likely to be as small as the sensed value. At best, the method would be of marginal usefulness and would limit satellite operation to the lower portion of the 560 - 800 km range. In spite of its admirable simplicity, the method does not seem to warrant further consideration at this time.

Magnetic Field of the Earth

The magnetic field of the earth conceivably can be used as a reference system for sensing attitude deviations. However, the orbit and altitude of the satellite bear no simple continuous relationship to the geometry of the magnetic field. This means that any magnetic field sensing method requires a time or position programing of the field expected under ideal altitude conditions. The difficulties resemble those of the inertial methods using angular acceleration measurements outlined in section 3.2. For example, there are difficulties in

1. Estimating the magnetic field along the orbit.
2. Mechanizing the programing of the expected field.
3. Checkpointing or monitoring to correct the time scale of the program.

Above all, there is the difficulty of magnetic field variations due to outside magnetic disturbances, which at present are of unknown nature and are, therefore, unpredictable.

Gravitational Field of the Earth

Among the methods of sensing on ambient fields, the most promising are those which make use of the differential gravitational forces on the various portions of the vehicle. Although the gravitational and centrifugal forces are balanced at the center of mass of the satellite, the gravitational force exceeds the centrifugal force below the center of mass, and conversely above. Thus,

an object below the center of mass has a net force along the inward-pointing normal of the local gravitational equipotential surface. If the direction of this force can be determined, and if the equipotential surface is approximately parallel to the earth's local tangent surface, then this small gravitational differential might permit a determination of the vertical desired for mapping purposes.

The approximate magnitude of the net force on such an object can be calculated readily. The force balances on a proof mass m at the center of mass and at a point P a distance a below it are shown in Fig. 10. For simplicity, it is presumed that the vehicle is in uniform motion with angular velocity $\dot{\theta}_2$ on a circular orbit of radius R, and that there are no rotational accelerations about the center of mass. The constant of universal gravitation is denoted by G, the mass of the earth by M_e. Using the fact that $\dot{\theta}_2^2$ is related to R by $\dot{\theta}_2^2 = M_e G / R^3$, the net downward force on the particle (hereafter spoken of as its "apparent weight") is

$$W_a = \frac{m M_e G}{(R-a)^2} - m(R-a)\dot{\theta}_2^2 = \frac{3 a m M_e G}{R^3} + 0\left(\frac{a^2}{R^2}\right) \tag{4}$$

Here $0(a^2/R^2)$ represents all residual terms of order greater than a/R in the power series expansion of W_a in a/R, which are negligible compared to the first order term in the present application.

Representing the earth weight of the particle by W_e and the mean earth radius by R_0, one has $m M_e G = m g R_0^2 = W_e R_0^2$, whence

$$W_a = \frac{3 a R_0^2}{R^3} W_e \tag{5}$$

Although this result does not take account of ellipticity in the orbit or earth, it suffices to indicate the order of magnitude of the forces involved.

With $R_0 = 6400$ km, and a expressed in meters, the coefficient of $a W_e$ in Eq. 5 is very close to 3×10^{-7} for $R - R_0$ in the range 560 - 800 km. Therefore, the apparent weight is essentially

$$W_a = 3 \times 10^{-7} a W_e \tag{6}$$

In mechanizing such a method successfully it is necessary only to determine the direction of a force whose magnitude is of the order of 10^{-3} newton (1 milligram force) as would be obtained with $a = 1$ and $W_e = 1$ kg (force). Techniques exist for doing this even in the presence of the full gravitational field of the earth, so that one may expect it to be possible, and even easier, under satellite conditions.

A description of a possible mechanization with a more complete discussion of the problems involved is given in Chapter 5.

Cosmic Rays

A method of determining the vertical by observations on the incident cosmic rays has been suggested (Ref. 13). Although this method may be applicable at sea level, no collimation of cosmic rays is to be expected at operating altitude. Any collimation observed on earth is caused by the atmosphere and the earth's magnetic field. In the absence of a preferential direction of incidence, the method is useless for satellite purposes.

3.4. METHODS OF SIGHTING ON CELESTIAL BODIES

Earth-Sighting by Map-Matching

One can imagine sensing attitude variations, particularly those of yaw, by comparing the view of the earth as seen by the satellite telescope with maps of the terrain over which it is passing. This technique resembles that of map-matching, which has found some use in navigation and location of targets. However, there are some significant difficulties which would have to be overcome. If the system is entirely vehicle-contained, it is necessary to carry complete maps of the earth's surface and to have a means of locating the instantaneous satellite position on these maps. There is no obvious simple method for doing this. The usual methods of map-matching are practical only because the course is known, and because the map area to be scanned can be kept relatively small. It is doubtful if a technique can be mechanized at all for regions within which there are no outstanding topographic features.

Earth-Sighting by Horizon-Scanning

If one cannot practically sense attitude deviations by observations on the fine structure of the earth's surface, there is still the possibility of sensing them by gross observations. As seen in the large from the satellite, the earth is a disk of somewhat different physical properties from the ambient space. Moreover, if the normal to this apparent disk is found, it is approximately alined with the normal to the local tangent plane of the earth ellipsoid. This suggests that the local vertical, the desired satellite nadir direction, can be determined by observations on the disk of the earth.

The method which has been proposed for doing this involves a scanning of the optical radiation coming to the vehicle from near the edge of the earth's disk. It is known that a radiation discontinuity exists across the horizon which can be detected by an appropriate sensitive cell and that the location of the horizon (i.e., disk edge) can be determined in this way. It is known that such

an infrared system is feasible for daylight operation and that if only daylight operation is desired it may be possible to improve the system by using visible or ultraviolet rather than infrared scanning. However, it is expected that a satisfactory mechanization for night use would be more difficult to obtain. A possible mechanization for a horizon-scanning system is described in Chapter 6.

Earth-Sighting with Beacons

One can imagine monitoring orientation by means of sightings by the vehicle on signals from ground beacons. For example, a beacon sends out a narrow beam signal in a direction normal to the vehicle orbit. As the vehicle passes abreast of the beacon, a signal from the ground instructs it to determine the direction (relative to the longitudinal axis) at which the signal is received. This direction is a measure of the instantaneous yaw angle. Although a number of variations on this theme can be conceived, they have certain disadvantages in common:

1. A large number of beacons is generally required.
2. Signals must be received by the vehicle from the ground, which is probably bad practice.
3. The vehicle position must be closely monitored by ground stations.
4. The inherent accuracy is not high.
5. Attitude checking is only intermittent.

Other methods seem preferable.

Earth-Sighting by Radar Reflection

One can imagine a method of determining the vertical by means of a radar beam reflected from the earth. If the beam is sent out along a line which revolves on a cone about the nadir direction, a return signal of equal strength is received at each azimuth when the vehicle is in its proper position relative to the vertical. If the zenith vehicle axis is inclined to the horizontal, however, there will be an azimuth direction at which a stronger signal is received. This will be the direction of the inclination of the vertical.

The following difficulties with this system are immediately apparent:

1. Relatively complex electronic equipment is required for mechanization.
2. Atmospheric dispersion may introduce serious errors.
3. The sensed vertical is strongly conditioned by local topographic features so that a long smoothing time is required to sense deviations of the vertical in positive fashion.
4. The signals may betray the location of the vehicle.

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Sun or Star Sightings

A continuous method of celestial navigation cannot be used by the satellite because there is no celestial body constantly in its view and because orbital unpredictability prevents any prior programing of sightings on several bodies. However, enough is known about the positions of the sun and Polaris as they rise and set (as seen from the satellite) that, if the orbital altitude and inclination are known, an intermittent system of attitude sensing can be based upon sightings on these bodies. There is no essential difference in principle between the two methods of sighting, although they differ in mechanization particulars. In star-sighting, a point target always appears at the same bearing in rising or setting. In sun-sighting, the bearing angle changes with time, but the target disk, although more ambiguously located because of its size, is more easily sensed by a sensitive element. The principle is described for sighting on Polaris, with the recognition that an analogous description holds for sun-sighting. In evaluating the latter method, it should be mentioned that sun-following devices have been developed by H. L. Clark at the Naval Research Laboratory and by W. P. Pietenpol at the University of Colorado.

As viewed from the satellite, Polaris follows a portion of a right cross section of an almost circular conical surface (Fig. 11) whose semivertex angle is γ (the orbital inclination to the equator) and whose axis is the port beam of the vehicle. The apparent motion of the star on this path is counterclockwise as seen from the vehicle. The small deviation of the cone of line-of-sight from a right circular cone, caused by the fact that Polaris is not precisely on the earth's polar axis, is neglected. It is presumed that the value of γ is great enough that Polaris rises and sets during each orbital revolution.

In its most rudimentary form, the method may be described for the case where the vertical is stabilized and where the only concern is determining yaw. In this case, if the orbital inclination γ to the equator is known, the yaw at star-rise can be determined by a single observation of the angle between the vehicle forward axis and the bearing to the star at the instant the star crosses the horizontal plane of the vehicle. However, to infer roll, pitch and yaw from a knowledge of the orbital inclination and the dip angle to the horizon (determined by the orbital attitude), is a somewhat more elaborate scheme.

Suppose that ξ, η, ζ are the axes of the local trihedral to the orbit, in the customary sense of differential geometry, except that ξ is the tangent direction, η the binormal, and ζ the negative of the usual normal. Further, suppose that the line-of-sight to Polaris describes a cone of semivertex angle γ about the η-axis. Then the bearing to the star may be described by two angles γ and α_1 (the inclination to the tangent plane), as shown in Fig. 12A. At the same time, the axes of the vehicle may be given relative to the local trihedral by yaw, pitch and roll angles θ_1, θ_2, θ_3, respectively, as shown

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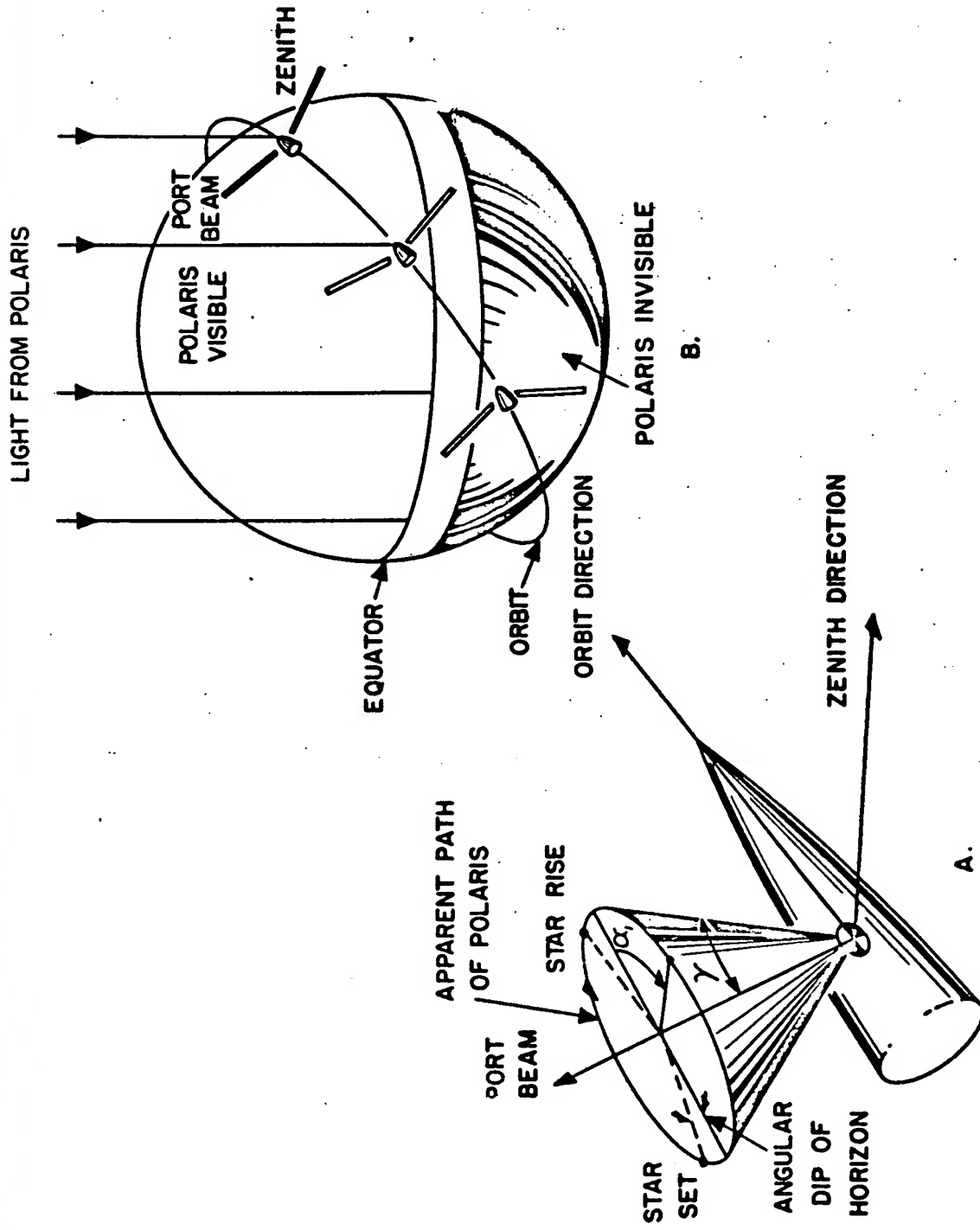
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Fig. 11. Apparent Path of Polaris Relative to Satellite

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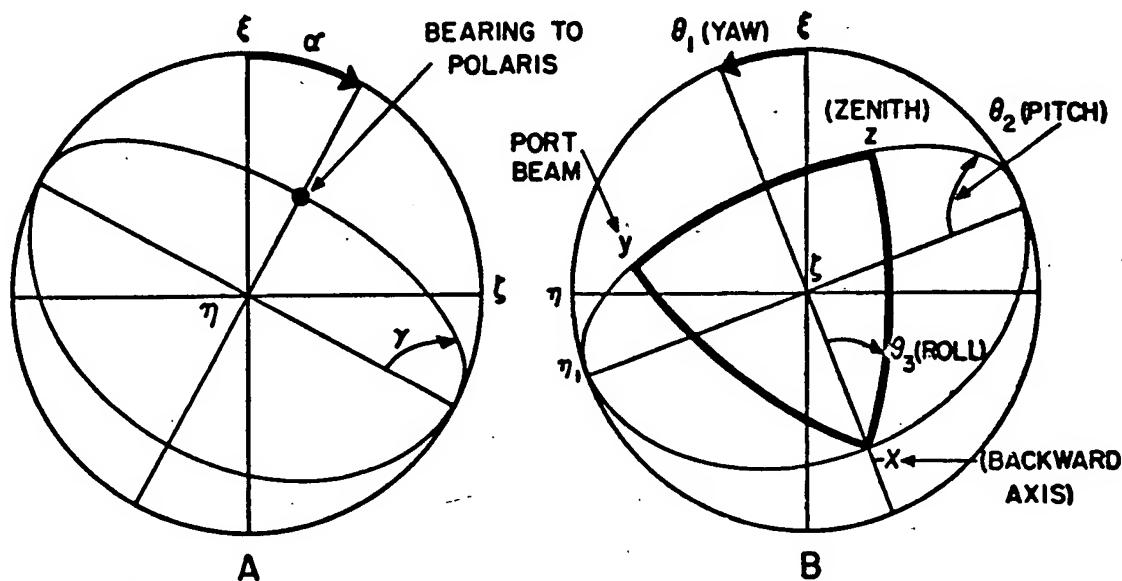
Fig. 12. Vehicle Orientation Relative to Orbital Local Trihedral

in Fig. 12B. It is easy to find the direction cosines of the bearing line relative to vehicle-fixed xyz axes. However, it is impossible to solve the resulting three equations for θ_1, θ_2, and θ_3, even when measured values of the direction cosines and a priori values of γ and α_1 are known; for, as the three direction cosines are not independent, the apparent three equations are actually only two in number.

It follows that one observation does not suffice. If two are made at convenient positions relative to the vehicle, the complication enters that the attitude deviations may not remain constant during the interval between observations. However, sufficient assumptions can be made to make the problem determinant. For example, one can assume that the attitude deviation rates do not change sensibly during the interval. Then, if three observations could be made with no appreciably greater complexity than two, six independent equations in the average values of the six variables $\theta_1, \theta_2, \theta_3, \dot{\theta}_1, \dot{\theta}_2,$ and $\dot{\theta}_3$ could be obtained and solved, giving values of these averages which maintain during a half-orbital period.

The principal disadvantages of this celestial navigation method are

1. It is relatively difficult to mechanize.
2. It requires a prior accurate knowledge of orbital inclination and altitude (or an additional system with which these can be inferred after the orbit is established).
3. It is not certain to work satisfactorily in the presence of scattered light near the horizon.
4. Refraction of rays near the horizon may introduce errors larger than can be permitted.

In all, it seems less feasible than the more promising of the methods described previously.

3.5. METHODS OF SENSING BY EARTH STATIONS

Systems of sensing can be developed in which the sensing is done by earth stations and an attitude correction signal is sent back to the vehicle. This class of systems is appealing, at least superficially, because it keeps all sensing equipment on the ground. Weight and power consumption are eliminated from the vehicle and the problem of long-time reliability of this equipment reduced. This is done at the expense only of adding provisions for receiving the correction signals and directing them to the control system.

As mentioned previously, it is doubtful whether any such system depending upon passive observation of the vehicle (e.g., by optical or radar observation) could be developed. Therefore, sensing must be done on signals from the vehicle. If special signals must be generated and transmitted, and provision must be made for reception of control signals, the system approaches the complexity of a completely vehicle-borne sensing system and probably is little more reliable. Therefore, one is led to consider sensing from the regularly televised pictures.

Here, there appear to be two difficulties. Suppose that the received picture is to be compared in orientation with a map of the terrain over which the vehicle is passing, with register between the pictures determined by a well-located check-point. Suppose further that a single received frame can be held as long as desired on a special device, to facilitate comparison. Then, first, there is still the likelihood of a time delay before the vehicle can be informed of its attitude errors, because the scanned frame must be adjusted and a series of mathematical operations performed in order to infer these errors. Second, there is the impasse that attitude adjustment is important while the vehicle is televising over enemy territory (i.e., in daylight), although the sensing can be done most conveniently over friendly territory, which is likely to be on the other side of the earth. But on the other side is night, where the satellite can not televise satisfactorily.

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To complete the list of difficulties, that of vulnerability should be given and stressed. Whenever a communication link is opened from ground to satellite, the vehicle is vulnerable to enemy countermeasures. In the absence of any other difficulty, this factor of vulnerability is serious enough to give pause to the adoption of a system of this class.

4. A YAW-SENSING GYRO

4.1. RESTRICTIONS

Some comments have been made in section 3.2 on the possibility of sensing yaw deviations by means of a properly mounted gyroscope. The details of a possible instrument of this kind are amplified in this chapter. The problem is not treated here in its full generality. The major restrictions are

1. No gimbal inertia.
2. No bearing friction.
3. Coincidence of the gyro center of mass, vehicle center of mass, and gimbal geometric center.
4. Consideration of only the linearized equations of motion.

The fourth restriction is more apparent than real, because a theorem of Liaponnoff (Ref. 14) states: "If the system of differential equations of the first approximation is regular, and if the characteristic numbers are positive, the unperturbed motion is stable." These conditions, equivalent to a requirement of stability in the ordinary sense, will be considered after the linearized equations are derived.

The first restriction merely reduces the complexity of the treatment without seriously limiting its generality, and the derivation is conducted in such a way that gimbal inertia can be introduced with minimum effort (as it is, later in this chapter). The second and third restrictions are partially compensated by the introduction of perturbation torques of unspecified origin acting on the gyro, but would have to be relaxed in a more elaborate error analysis.

One further restriction is made after the equations are derived, but before their interpretation is carried through. This restriction is that the control torques be such that roll and pitch attitudes are kept identically equal to zero. Actually, these torques will be prescribed by the sensing system which determines the vertical, and eventually the combined yaw and vertical sensing and control systems will have to be treated. For this reason, the initial derivation is conducted with these angles explicit.

More important than these restrictions, however, is the fact that the behavior of the vehicle on the orbit is taken fully into account, and that no restrictions are made to a plane circular orbit. In the latter case, one is sure that such an instrument can be made to work, as it then becomes a conventionally applied gyrocompass. The treatment is considerably complicated by

this inclusion, which introduces variable coefficients into the equations of motion. Therefore, it is not possible to obtain the complete explicit analytical results which may be desired.

It is considered that the desired vehicle attitude places the principal axes along the local trihedral to the orbital curve, the latter being described by two basic parameters ω_1 and ω_2. However, if it is desired that the pitch rate be constant, this case can be obtained by putting $\dot{\omega}_2 = 0$. Other desired attitude-position or attitude-time relationships can be realized by choosing ω_1 and ω_2 as appropriate functions of position or time, rather than as their "natural" values which describe the orbital curve.

A schematic drawing of the yaw sensing instrument is given in Fig. 13. The xyz coordinate system is vehicle-fixed, and is related to the local trihedral for the vehicle trajectory as shown in Fig. 12B. The xyz axes are respectively roll, pitch and yaw axes. Figure 14 shows the relation of gyro and gimbal rings to the vehicle in such a way as to define angles α and β and certain additional coordinate axes convenient for use.
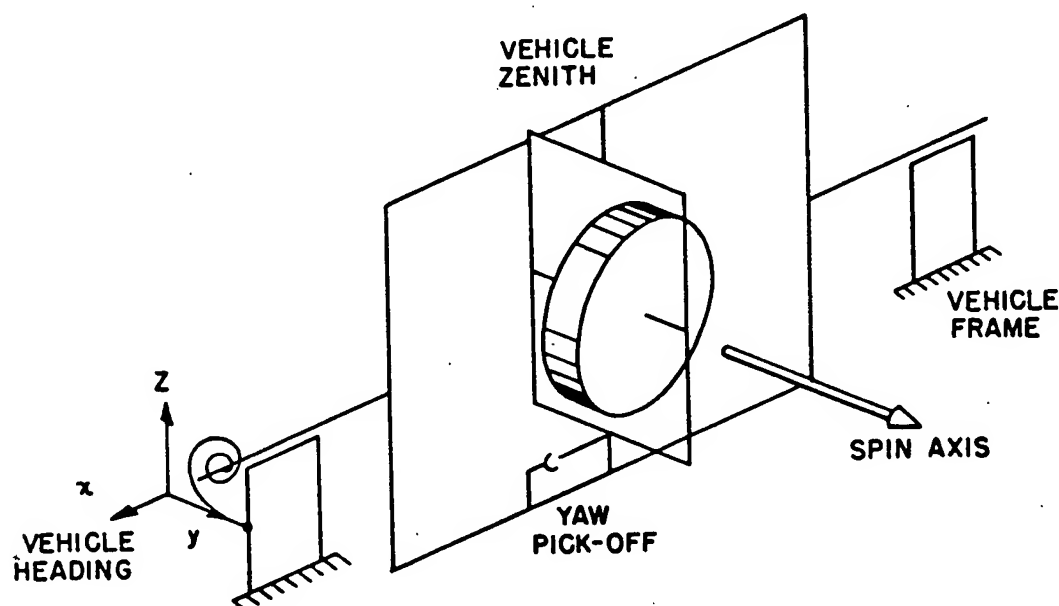
Fig. 13. Schematic Diagram of Yaw Gyro
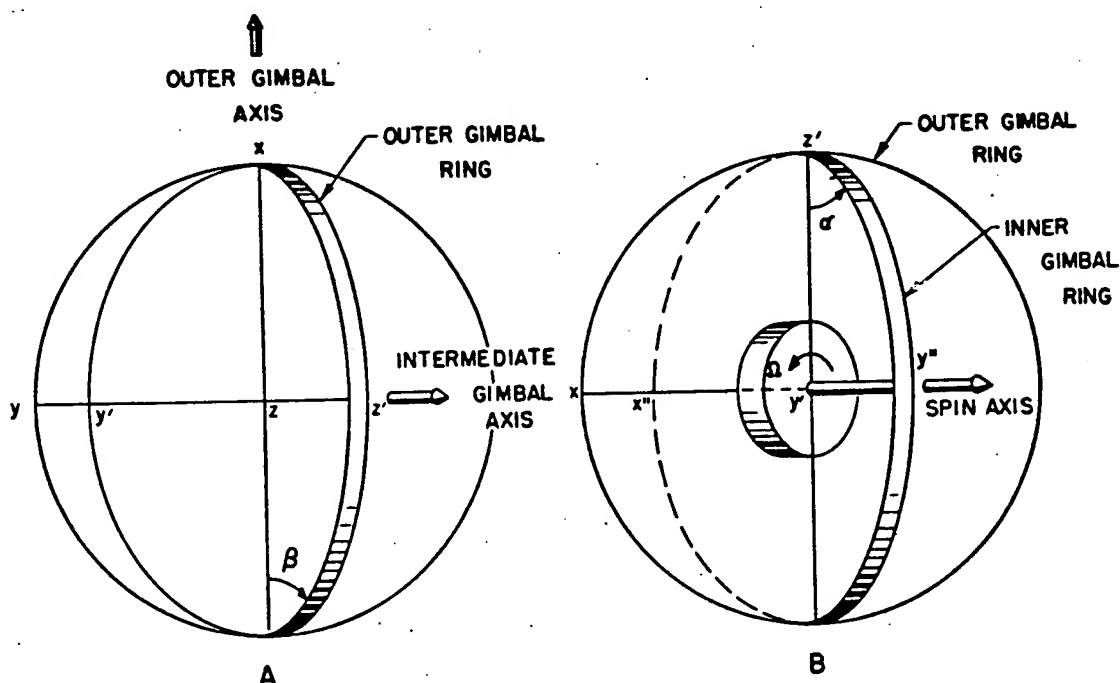
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Fig. 14. Yaw Gyro and Gimbal Rings in Relation to Vehicle-Fixed xyz Axes
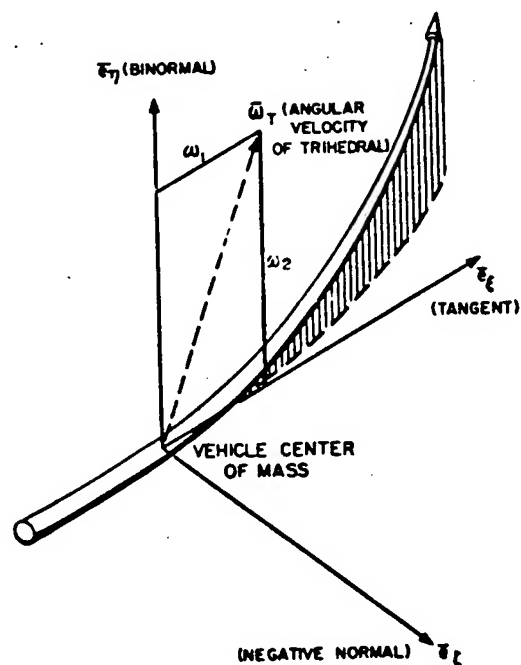
Fig. 15. Angular Velocity of Local Trihedral of Orbit

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4.2. GENERAL LINEARIZED EQUATIONS

For the Vehicle

If v is the path speed of the vehicle, $1/\tau$ the orbital torsion, and $1/\rho$ the orbital curvature, one may define

$$\omega_1 = v/\tau \text{ and } \omega_2 = v/\rho \tag{7}$$

Denote the unit vector along any axis q by $\bar{e}_q$. It can be shown (Ref. 15) that the angular velocity in space of the local trihedral is $\omega_1\bar{e}_\xi + \omega_2\bar{e}_\eta$ (Fig. 15). Then the angular velocity of the vehicle in space is

$$\bar{\omega}_\gamma = \dot{\theta}_1\bar{e}_\zeta + \dot{\theta}_2\bar{e}_{\eta'} + \dot{\theta}_3\bar{e}_x + \omega_1\bar{e}_\xi + \omega_2\bar{e}_\eta \tag{8}$$

Referring this velocity to vehicle-fixed axes and discarding second order terms,

$$\bar{\omega}_\gamma = [\dot{\theta}_3 + \omega_2\theta_1 + \omega_1]\bar{e}_x + [\dot{\theta}_2 - \omega_1\theta_1 + \omega_2]\bar{e}_y + [\dot{\theta}_1 + \omega_1\theta_2 - \omega_2\theta_3]\bar{e}_z \tag{9}$$

Because x, y, and z are principal axes, the inertia tensor of the vehicle can be written

$$I_\gamma\bar{e}_x\bar{e}_x + J_\gamma\bar{e}_y\bar{e}_y + K_\gamma\bar{e}_z\bar{e}_z \tag{10}$$

Also, the total torque acting on the vehicle may be decomposed into two components: L, the externally applied torque arising from the control system; and $L_{\gamma R}$, the reaction torque exerted on the vehicle by the gyro gimbal ring. At first, the external torques will just be considered as given a priori, although later it may be supposed that a control system is introduced which makes one or more of the applied torques a function of the dependent variables of the sensing systems.

Using Eq. 9 and 10, the Euler equations of motion of the vehicle may be written at once. In linearized form, they are

$$I_\gamma\ddot{\theta}_3 + (K_\gamma - J_\gamma + I_\gamma)\omega_2\dot{\theta}_1 + I_\gamma\dot{\omega}_2\theta_1 + (K_\gamma - J_\gamma)\omega_1\omega_2\theta_2$$

$$-(K_\gamma - J_\gamma)\omega_2^2\theta_3 = L_x - I_\gamma\dot{\omega}_1 + (L_{\gamma R})_x \tag{11}$$

$$J_r \ddot{\theta}_2 + (I_r - K_r - J_r)\omega_1 \dot{\theta}_1 - J_r \dot{\omega}_1 \theta_1 + (I_r - K_r)\omega_1^2 \theta_2 - (I_r - K_r)\omega_1 \omega_2 \theta_3$$

$$= L_y - J_r \dot{\omega}_2 + (L_{rR})_y \qquad (12)$$

$$K_r \ddot{\theta}_1 + (J_r - I_r + K_r)\omega_1 \dot{\theta}_2 + (J_r - I_r - K_r)\omega_2 \dot{\theta}_3 + (J_r - I_r)(\omega_2^2 - \omega_1^2)\theta_1$$

$$+ K_r \dot{\omega}_1 \theta_2 - K_r \dot{\omega}_2 \theta_3 = L_z - \omega_1 \omega_2 (J_r - I_r) + (L_{rR})_z \qquad (13)$$

For the Gyro

It is evident from Fig. 13 that the angular velocity in space of the gyro is
is

$$\bar{\omega}_r + \dot{\beta}\bar{e}_x + \Omega \bar{e}_{y'} + \dot{\alpha}\bar{e}_{z'}$$

$$= [\dot{\theta}_3 + \dot{\beta} + \omega_2 \theta_1 + \omega_2 \alpha + \omega_1]\bar{e}_{x'} + [\dot{\theta}_2 - \omega_1 \theta_1 - \omega_1 \alpha + \omega_2 + \Omega]\bar{e}_{y'}$$

$$\qquad (14)$$

$$+ [\dot{\theta}_1 + \dot{\alpha} + \omega_1 \theta_2 - \omega_2 \theta_3 - \omega_2 \beta]\bar{e}_{z'}$$

However, the angular velocity of the $x''y''z'$ coordinate system is less than
this by an amount $\Omega \bar{e}_{y''}$, and the Euler equations must be modified accordingly.
The inertia tensor is

$$I_G \bar{e}_{x''}\bar{e}_{x''} + J_G \bar{e}_{y''}\bar{e}_{y''} + I_G \bar{e}_{z''}\bar{e}_{z''} \qquad (15)$$

where the axial symmetry of the gyro rotor about the y'' axis already has been
used. Denote the torque applied to the gyro by the gimbal rings by $\bar{L}_{GR}$. In
accordance with the remarks of section 4.1, it is assumed that there is an ad-
ditional perturbation torque P acting on the gyro from various unspecified
sources. It is presumed, however, that these torques do not react back on the
vehicle. Actually, any such reaction torques are probably small compared with
the external control torques.

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The equations of motion of the gyro are

$$I_G(\ddot{\theta}_3 + \ddot{\beta}) + [(2I_G - J_G)\omega_2 - J_G\Omega](\dot{\theta}_1 + \alpha) + I_G\dot{\omega}_1(\theta_1 + \alpha) + [(I_G - J_G)\omega_2 - J_G\Omega][\omega_1\theta_2$$
$$-\omega_2(\theta_3 + \beta)] = (L_{GR})_{x^*} - I_G\dot{\omega}_1 + P_{x^*} \tag{16}$$

$$J_G[\dot{\Omega} + \ddot{\theta}_2 - \omega_1(\dot{\theta}_1 + \dot{\alpha}) - \dot{\omega}_1(\theta_1 + \alpha)] = (L_{GR})_{y^*} - J_G\dot{\omega}_2 + P_{y^*} \tag{17}$$

$$I_G(\ddot{\theta}_1 + \ddot{\alpha}) + J_G\omega_1\dot{\theta}_2 + [(J_G - 2I_G)\omega_2 + \Omega J_G](\dot{\theta}_3 + \dot{\beta}) + [(J_G - I_G)(\omega_2{}^2 - \omega_1{}^2)$$

$$+ J_G\Omega\omega_2](\theta_1 + \alpha) + I_G\dot{\omega}_1\theta_2 - I_G\dot{\omega}_2(\theta_3 + \beta) = \tag{18}$$

$$(L_{GR})_{z^*} - [(J_G - I_G)\omega_2 + J_G\Omega]\omega_1$$

For the Gimbal Rings

In this preliminary study it is supposed that the gimbal rings are inertialess. Although their inertia may modify the results slightly, the essential features of the yaw-sensing gyro can be brought out without the complications of this additional analysis. Under these conditions, there are no inertial torques corresponding to the gimbal ring motion, and one has immediately the relation

$$\bar{L}_{YR} = -\bar{L}_{GR} \tag{19}$$

Some Torque Conditions

At this point, six equations of motion for the system have been obtained. However, these involve nine unknown variables, even if the information of Eq. 19 is used to remove certain unknown torques. The nine may be considered to be $\theta_1, \theta_2, \theta_3, \alpha, \beta, \Omega, (L_{GR})_{x^*}, (L_{GR})_{y^*}$, and $(L_{GR})_{z^*}$. In order to make the system determinant, three additional conditions must be given. It is these conditions, moreover, which characterize the particular instrument at hand as distinct from other possible gyro systems of the same general form.

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The following conditions are adjoined:

1. $(L_{GR})_{y'}$ is such that $\dot{\Omega} = 0$, as would be closely the case with a synchronous motor.

2. $(L_{GR})_{z'} = 0$, implying that there is a low coercion bearing along this axis.

3. $(L_{VR})_x = f(\beta, \dot{\beta})$, implying a spring and damper coupling between the outer gimbal ring and the vehicle frame.

If these conditions are used in Eq. 11 to 13 and 16 to 18, the interaction torques which are of no consequence can be eliminated and one can obtain a complete and independent set of five equations (in which Ω is a parameter). Making a simplification by noting that $\Omega \gg \omega_1$, ω_2, these equations can be written:

$$K_V \ddot{\theta}_1 + (J_V - I_V - K_V)\omega_1 \dot{\theta}_2 + (J_V - I_V - K_V)\omega_2 \dot{\theta}_3 + (J_V - I_V)(\omega_2{}^2 - \omega_1{}^2)\theta_1$$

$$+ K_V \dot{\omega}_1 \theta_2 - K_V \dot{\omega}_2 \theta_3 = L_z - \omega_1 \omega_2 (J_V - I_V) - \beta J_G \dot{\omega}_2 \tag{20}$$

$$J \ddot{\theta}_2 + (I - K - J)\omega_1 \dot{\theta}_1 - J_G \omega_1 \dot{\alpha} - J \dot{\omega}_1 \theta_1 + (I_G - J_G)\dot{\omega}_1 \alpha$$

$$+ (I_V - K_V)\omega_1{}^2 \theta_2 - (I_V - K_V)\omega_1 \omega_2 \theta_3 = L_y - J \dot{\omega}_2 + P_{y'} \tag{21}$$

$$I_V \ddot{\theta}_3 + (K_V - J_V + I_V)\omega_2 \dot{\theta}_1 + I_V \dot{\omega}_2 \theta_1 + (K_V - J_V)\omega_1 \omega_2 \theta_2$$

$$- (K_V - J_V)\omega_2{}^2 \theta_3 = L_x - I_V \dot{\omega}_1 + f(\beta, \dot{\beta}) \tag{22}$$

$$I_G(\ddot{\theta}_3 + \ddot{\beta}) - J_G \Omega(\dot{\theta}_1 + \dot{\alpha}) + I_G \dot{\omega}_1(\theta_1 + \alpha) - J_G \Omega \omega_1 \theta_2$$

$$+ J_G \Omega \omega_2(\theta_3 + \beta) = -f(\beta, \dot{\beta}) + J_G \dot{\omega}_2 \alpha - I_G \dot{\omega}_1 + P_{x'} \tag{23}$$

$$I_G(\ddot{\theta}_1 + \ddot{\alpha}) + J_G \omega_1 \dot{\theta}_2 + \Omega J_G(\dot{\theta}_3 + \dot{\beta}) + J_G \Omega \omega_2(\theta_1 + \alpha)$$

$$+ I_G \dot{\omega}_1 \theta_2 - I_G \dot{\omega}_2(\theta_3 + \beta) = -J_G \Omega \omega_1 + P_{z'} \tag{24}$$

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where

$$I = I_{\nu} + I_{G}, \; J = J_{\nu} + J_{G}, \;\; K = K_{\nu} + K_{G} \qquad (25)$$

Finally, if one supposes that L_x, L_y are of just the right magnitudes to keep $\theta_2 = \theta_3 = 0$,

and puts

$$f(\beta, \dot{\beta}) = F\beta + B\dot{\beta} \qquad (26)$$

then

$$K_{\nu}\ddot{\theta}_1 + (J_{\nu} - I_{\nu})(\omega_2^2 - \omega_1^2)\theta_1 = L_z - \omega_1\omega_2(J_{\nu} - I_{\nu}) \qquad (27)$$

$$I_{G}\ddot{\beta} - J_{G}\Omega(\dot{\theta}_1 + \dot{\alpha}) + I_{G}\dot{\omega}_1(\theta_1 + \alpha) + (J_{G}\Omega\omega_2 + F)\beta$$
$$+ B\dot{\beta} - J_{G}\dot{\omega}_2\alpha = P_x \cdot - I_{G}\dot{\omega}_1 \qquad (28)$$

$$I_{G}(\ddot{\theta}_1 + \ddot{\alpha}) + J_{G}\Omega\dot{\beta} + J_{G}\Omega\omega_2(\theta_1 + \alpha) - I_{G}\dot{\omega}_2\beta = P_z \cdot - J_{G}\Omega\omega_1 \qquad (29)$$

At this point, it should be recalled that L_z can be made a function of the sensed angle α. In particular, a convenient control might be

$$L_z = \omega_1\omega_2(J_{\nu} - I_{\nu}) + C\alpha \qquad (30)$$

with C a constant whose value is yet unspecified.

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4.3 GYRO BEHAVIOR FOR SPECIAL ORBITS

It is instructive to consider certain special cases characterized by the behavior of ω_1, ω_2. The equations of motion which are used for these cases are Eq. 27 and 30, with

$$\phi = \theta_1 + \alpha \tag{31}$$

Orbit with Constant Curvature, Zero Torsion

This is a plane, circular orbit for which $\omega_1 = 0$, $\dot{\omega}_2 = 0$ $(\omega_2 \neq 0)$. The equations of motion are

$$K_\gamma \ddot{\theta}_1 + (J_\gamma - K_\gamma)\omega_2^2 \theta_1 = C\alpha \tag{32}$$

$$I_G \ddot{\beta} - J_G \Omega \dot{\phi} + B\dot{\beta} + (F + J_G \Omega \omega_2)\beta = P_{x'} \tag{33}$$

$$I_G \ddot{\phi} + J_G \Omega \omega_2 \phi + J_G \Omega \dot{\beta} = P_{z'} \tag{34}$$

By virtue of the disappearance of the lone α-term in Eq. 33, the set of equations separates. The second and third of these equations do not involve θ_1, and may be solved simultaneously for β and ϕ. The result may be used in Eq. 32 to find θ_1.

A single equation for ϕ may be obtained from Eq. 32 to 34. Denoting the appropriate combination of the perturbation torques by Π,

$$\ddddot{\phi} + \frac{B}{I_G} \dddot{\phi} + \left(\frac{F}{I_G} + 2\frac{J_G \Omega \omega_2}{I_G} + \frac{J_G^2 \Omega^2}{I_G^2} \right) \ddot{\phi} + \frac{B J_G \Omega \omega_2}{I_G^2} \dot{\phi}$$

$$\tag{35}$$

$$+ \frac{J_G \Omega \omega_2}{I_G} \left(\frac{F + J_G \Omega \omega_2}{I_G} \right) \phi = \Pi$$

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The roots of the corresponding characteristic equation have only negative real parts, so that the motion is stable. The steady-state value of ϕ will not be zero in general, nor will that of β. However, their magnitudes will be governed by the drift-causing perturbation torques and by the design parameters of the system, and probably will be small.

It is not appropriate to treat the details of a system design involving a choice of B, C, F at this point, as this case is not the one realized in practice.

Orbit with Constant Curvature, Constant Torsion

This would be the case of a right circular helical orbit, which again is unrealistic. However, it is interesting to see the way in which this case differs from the previous one. The equations are

$$K_\gamma \ddot{\theta}_1 + (J_\gamma - I_\gamma)(\omega_2{}^2 - \omega_1{}^2)\theta_1 = C\alpha \tag{36}$$

$$I_G \ddot{\beta} - J_G \Omega \dot{\phi} + (F + J_G \Omega \omega_2)\beta + B\dot{\beta} = P_{x''} \tag{37}$$

$$I_G \ddot{\phi} + J_G \Omega \dot{\beta} + J_G \Omega \omega_2 \phi = P_{z'} - J_G \Omega \omega_1 \tag{38}$$

It is seen that the only modifications of Eq. 32 to 34 are in the slight change of natural frequency in the θ_1 equation, and in the introduction of an additional perturbation torque with $P_{z'}$.

If $P_{z'}$ is constant and $P_{x''}$ is zero, then the steady state error in ϕ will be

$$\phi(\infty) = \frac{I_G P_{z'}}{J_G \Omega \omega_2} - \frac{\omega_1}{\omega_2} \tag{39}$$

as compared with the corresponding

$$\phi(\infty) = \frac{I_G P_{z'}}{J_G \Omega \omega_2} \tag{40}$$

for the case of zero torsion.

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Some feel for the numerical quantities involved may be obtained by supposing the helix has a radius cf 6400 km and a pitch of 30 km (which is the approximate displacement of the line of modes per orbital revolution in the actual case).   Then it can be shown that the ratio ω_1/ω_2 will be

$$\omega_1/\omega_2 = 30/2\pi(6400) \approx 0.75 \text{ milliradian}$$

Therefore, one sees that the change in attitude produced by the effect of a constant orbital torsion on the instrument is likely to be entirely insignificant for the intended use of the vehicle.   It will probably be even less important in the presence of errors caused by any perturbation torque that might exist.

Orbit with Constant Curvature, Constant Rate of Torsion

The equations of motion for this case are

$$K_\gamma \ddot{\theta}_1 + [(J_\gamma - I_\gamma)(\omega_2{}^2 - \omega_1{}^2) + C]\theta_1 = C\phi \tag{41}$$

$$I_G\ddot{\beta} + B\dot{\beta} + (F + J_G\Omega\omega_2)\beta - J_G\Omega\dot{\phi} + I_G\dot{\omega}_1\phi = (P_{x''} - I_G\dot{\omega}_1) \tag{42}$$

$$I_G\ddot{\phi} + J_G\Omega\omega_2\phi + J_G\Omega\dot{\beta} = (P_{z'} - J_G\Omega\omega_1) \tag{43}$$

These lead to the equation for ϕ:

$$\ddddot{\phi} + \frac{B}{I_G}\dddot{\phi} + \left(\frac{F}{I_G} + \frac{2J_G\Omega\omega_2}{I_G} + \frac{J_G{}^2\Omega^2}{I_G{}^2}\right)\ddot{\phi} + \frac{BJ_G\Omega\omega_2}{I_G{}^2}\left(1 - \frac{\dot{\omega}_1 I_G}{\omega_2 B}\right)\dot{\phi}$$

$$+ \frac{J_G\Omega\omega_2}{I_G}\left(\frac{F + J_G\Omega\omega_2}{I_G}\right)\phi = \Pi + \frac{(F + J_G\Omega\omega_2)}{I_G{}^2}\left(P_{z'} - J_G\Omega\omega_1\right) - \frac{BJ_G\Omega\dot{\omega}_1}{I_G} \tag{44}$$

This is an interesting result, as it assures that the only modifications of the present case from the case of a plane orbit are the occurrence of the factor $(1 - \dot{\omega}_1 I_G/\omega_2 B)$ instead of unity in the coefficient of $\dot{\phi}$, and the addition of a perturbation term to $P_{z'}$.   It is easy to be assured that the former is not significant. Suppose that $\dot{\omega}_1/\omega_2 = 60/2\pi(6400)(6000 \text{ sec})$, so that the average ω_1

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over an orbital half-period is the same value as used in the second case. Reasonable physical values for I_0 and B are 5×10^{-4} kg-m^2 and 1 newton-m-sec respectively, so that $\dot{\omega}_1 I_0 / \omega_2 B$ is of the order of $10^{-9} \ll 1$.

Of the perturbation terms, that in $\omega_1(t)$ corresponds to an increasing excitation of the system, and will be considered only in the following section. That in ω_1 represents a constant offset or error in ϕ of magnitude $\dot{\omega}_1 I_0 / \omega_2 (F + J_0 \Omega \omega_2)$. For reasonable values of the parameter, this is of the order of 10^{-7} radian and may be neglected summarily.

Orbit with Constant Curvature, Sawtooth Torsion

A more realistic torsion angular velocity can now be considered, with the aid of the result preceding. The actual torsion is periodic, with period equal to the orbital period, although the exact functional form of the torsion is not yet known. However, it seems likely that the first Fourier component of this function can be made to dominate in determining the behavior of the yaw gyro, by an appropriate choice of gyro parameters. Therefore, it is proposed to examine this behavior by replacing the actual $\omega_1(t)$ function by a sawtooth, piecewise linear, function with an amplitude so chosen that its first Fourier component is the same as that of the actual torsion angular velocity. This sawtooth function is shown in Fig. 16. Over any quarter period within which $\omega_1(t)$ is linearly increasing or decreasing, one can use the results of the previous case.
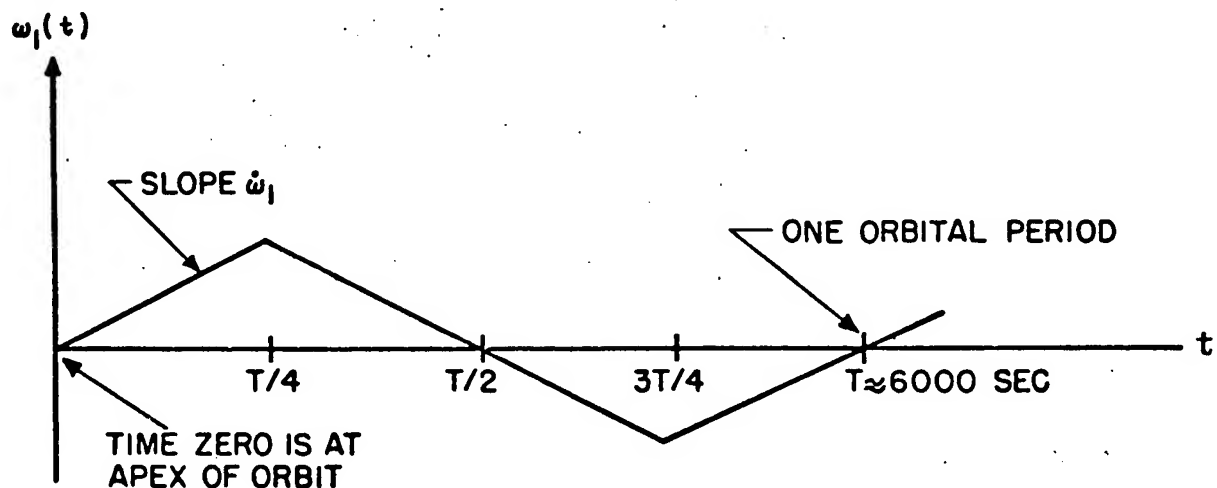
Fig. 16. Approximation to Orbital Torsion

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Because the system is stable, the initial conditions damp out eventually, and only the periodic response to $\omega_1(t)$ need be considered. Provided the natural frequencies of the system are considerably smaller than the orbital frequency, it can be shown that the first Fourier coefficient of the response is approximately $-\mathcal{F}_1/\omega_2$, where $\mathcal{F}_1$ is the first Fourier coefficient of $\omega_1(t)$. It has already been seen that this ratio is of the order of 1 milliradian for the torsion and curvature magnitudes under consideration. One therefore concludes that, although there is a small periodic response $\phi(t)$ to the torsion driving torque, its amplitude is of the order of only a milliradian. This is probably of the order of, or smaller than, the response to the perturbation torques Π.

Orbit with Constant Rates of Curvature and Torsion

A somewhat more general case can be built up by permitting ω_2 to vary linearly, at least during a restricted time range. It can be shown that if β is eliminated between Eq. 28 and 29, and if one disregards coefficients involving $\dot{\omega}_1$, $\dot{\omega}_2$, and $\dot{\omega}_2^2$, which are small compared to unity, then the following equation results:

$$
\overset{....}{\phi} + \frac{B}{I_G}\overset{...}{\phi} + \left(\frac{F}{I_G} + \frac{2J_G\Omega\omega_2}{I_G} + \frac{J_G^2\Omega^2}{I_G^2}\right)\overset{..}{\phi} + \frac{BJ_G\Omega\omega_2}{I_G^2}\overset{.}{\phi}
$$

$$
+ \frac{J_G\Omega\omega_2}{I_G}\left(\frac{F + J_G\Omega\dot{\omega}_2}{I_G}\right)\phi = \Pi - \frac{J_G\Omega\omega_1}{I_G^2}(F + J_G\Omega\omega_2) \tag{45}
$$

$$
- \frac{J_G I_G \dot{\omega}_2^2}{(F + J_G\Omega\omega_2)}\left(\frac{J_G\Omega}{I_G}\right)^2 \theta_1
$$

To within these approximations, the only differences from earlier cases are the very small changes in some of the coefficients which take place during the time interval under consideration, and the introduction of a θ_1 coupling term between this equation in ϕ and the equation relating ϕ and ϕ_1 (of the form of Eq. 41). It can be shown that this leads to an equation for stable motion, and that the static error in ϕ is of the same order as in the previous cases.

Summary of Orbital Effects and Errors

The general response of the gyro system to changing orbital curvature and torsion has not been given. If the form of the curvature and torsion functions were known, an approximate method analogous to the Duffing method

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for nonlinear vibrations would be expected to give estimates of the periodic system response amplitudes satisfactorily close to their true values. In the absence of such knowledge, the system has been examined for certain special orbits corresponding to constant curvature and torsion and constant rates of these quantities. An argument for the extension of these simple cases to approximate the true conditions has led to an estimated angular error from this source of the order of 1 milliradian. This means, in effect, that the instrument can be treated in its design as if it were going to be carried around a plane circular path.

It can be seen from Eq. 45 that the introduction of a changing curvature does not introduce additional perturbation torques into the system. This is to be expected, because if the gyro axis is alined with the instantaneous pitch axis, as is almost the case when the instrument is in its normal operating configuration, rotation of the vehicle at any rate about its axis applies no torque tending to change the orientation of the gyro to its spin axis. It is corollary that a small θ_2 misalinement of the vehicle will not cause a significant error in the indicated yaw. On the other hand, a θ_3 roll misalinement of the vehicle will be reflected by a comparable gyro position error.

The principal errors in yaw, therefore, seem to be those caused by random perturbation torques and by roll misalinement. Errors due to orbital curvature and torsion are probably much smaller. It does not seem likely that total errors can be reduced to the order of one milliradian by any reasonable means. However, improvements in accuracy can be expected both by reduction of the perturbations and by improvements in the sensing and control of the vehicle zenith direction.

4.4. DESIGN PARAMETERS

Equations for the Physical Instrument

In discussing the selection of design parameters for the physical instrument, it is convenient to change slightly the point of view which is adopted toward the instrument. By the previous remarks, it is permissible to consider only motion in a plane circular orbit. Suppose, as before, that roll and pitch are perfectly controlled. Now consider, however, that proper control torques have been applied to bring the gyro angle α to zero. Also consider that the gimbal rings have inertia, so that I_θ is the moment of inertia of the inner gimbal and gyro wheel about the z' axis, and I_θ is the moment of inertia of the gyro wheel and both inner and outer gimbal rings about the x-axis.

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Then Eq. 32 to 34 reduce to the set

$$K_G \ddot{\theta}_1 + J_G \Omega \omega_2 \theta_1 + J_G \Omega \dot{\beta} = P_z,$$

(46)

$$I_G \ddot{\beta} + B\dot{\beta} + (F + J_G \Omega \omega_2)\beta - J_G \Omega \dot{\theta}_1 = P_x$$

(47)

The variable θ_1 can be eliminated from these equations to obtain

$$\dddot{\beta} + \frac{B}{I_G} \dddot{\beta} + \left(\frac{F}{I_G} + \frac{J_G \Omega \omega_2}{I_G} + \frac{J_G \Omega \omega_2}{K_G} + \frac{J_G^2 \Omega^2}{I_G K_G} \right) \ddot{\beta} + \frac{B J_G \Omega}{I_G K_G} \dot{\beta}$$

$$+ (F + J_G \Omega \omega_2) \frac{J_G \Omega \omega_2}{I_G K_G} \beta = \left(\frac{J_G \Omega \omega_2}{I_G K_G} P_x \frac{J_G \Omega}{I_G K_G} \dot{P}_z, + \frac{1}{I_G} \ddot{P}_z \right)$$

(48)

The left hand side of this equation is very nearly equal to

$$\left[\frac{d^2}{dt^2} + \frac{B}{I_G} \frac{d}{dt} + \frac{FK_G + J_G^2 \Omega^2}{I_G K_G} \right] \left[\frac{d^2}{dt^2} + \frac{B\omega_2}{J_G \Omega} \frac{d}{dt} + \frac{\omega_2(F + J_G \Omega \omega_2)}{J_G \Omega} \right] \beta$$

(49)

for physically reasonable parameter values.

Interpretation of Expression 49

The motion described by the factored operator of expression 49 consists of two independent damped oscillations. Certain coupling terms between these modes of vibration have been discarded in going from Eq. 48 to expression 49. However, these are sufficiently small that no appreciable energy transfer between modes can occur before the motions in the separate modes have each been damped out.

The vibrational mode represented by the first bracket of expression 49 has a natural frequency

$$\sqrt{(FK_G + J_G^2 \Omega^2)/K_G I_G}$$

and is critically damped when

$$B = 2\sqrt{(FK_G + J_G^2 \Omega^2)I_G/K_G} \, , \tag{50}$$

It is independent of the orbital angular velocity ω_2, and represents the natural nutational mode of the gyro and gimbal system which would maintain, by virtue of its physical characteristics above, if the vehicle frame were fixed in inertial space.

The second factor of that expression represents a vibration which is definitely associated with the orbital motion, as expressed by the presence of ω_2 in this factor. Because this oscillation is characteristic of the use of the instrument as a gyrocompass, it may be called the "gyrocompass oscillation" of the instrument. Its natural frequency is

$$\sqrt{\omega_2^2 + F\omega_2/J_G \Omega}$$

and it is critically damped when

$$B = 2\sqrt{J_G^2 \Omega^2 + FJ_G \Omega/\omega_2} \tag{51}$$

(It is not to be inferred that either of these critical damping values is necessarily optimum for B).

The static error for β can be found for the case of a constant perturbation torque from Eq. 48. The corresponding vehicle heading error is

$$|\theta_1| = |P_z|/J_G \Omega \omega_2 \tag{52}$$

Alternately, an expected heading error could be obtained for nonconstant random perturbation torques whose statistical characteristics are known. Equation 52 can be reformulated in terms of the gyro drift rate $|P_z|/J_G \Omega$ as

$$|\theta_1| = \frac{1}{\omega_2} \times \text{drift rate} \tag{53}$$

If the angular error in heading is not to exceed 1 milliradian, it follows that the gyro must have a drift rate not exceeding 10^{-6} radian/sec (0.2 deg/hr).

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Instrument Design Parameters

Optimum parameters must be chosen in view of the over-all sensing and control system requirements. This selection will require further study of the system as a whole. However, some notion of reasonable (order of magnitude) values for the several parameters can be obtained by making a few assumptions about these requirements. The parameters to be selected are the gyro angular momentum $H = J_G \Omega$, the spring constants F and B, the moments of inertia I_G and K_G, and the wheel speed Ω.

A subjective estimate of the wheel speed low enough to insure the long life of the spin bearings (which might be lightly loaded ball bearings) is $\Omega = 100$ rps = 630 radian/sec. Assume that a wheel angular momentum 10^{-1} newton-meter-sec (10^6 dyne-cm-sec) suffices to insure that the vehicle heading will be in error under the expected perturbation torques by no more than a few milliradians. For this a value of $J_G = 1.6 \times 10^{-4}$ kg-m^2 (1600 gm-cm^2) is required as the polar moment of inertia of the gyro wheel. This could be obtained with a wheel weighing approximately 0.25 kg with a diameter of 0.0635 m (2.5 in.). Operating power for such a gyro would be less than 5 w.

Suppose that the gimbal bearings are torsion wires backed up by a radial overload ring. Representative moments of inertia with this construction are $I_G = 4 \times 10^{-4}$ kg-m^2 (4000 gm-cm^2) and $K_G = 2.5 \times 10^{-4}$ kg-m^2 (2500 gm-cm^2).

Suppose that a desirable frequency for the gyrocompass oscillations is of the order of $5\omega_2$. It follows that the spring constant F should be $F = 24\omega_2 J_G \Omega$ or, numerically, $F = 2.4 \times 10^{-3}$ newton-meter/radian (24 dyne-cm/milliradian). The corresponding nutation frequency can then be calculated, and can be shown to be 316 radian/sec (50 cps). If the gyrocompass oscillations are to be critically damped by a choice of B, one obtains $B = 1$ newton-meter-sec (10^7 dyne-cm-sec). It follows that the nutation damping is eight times its critical value.

It is interesting to check the factorization of the quadratic form of Eq. 49, using the numerical parameter values which have been obtained. The unfactored form is

$$\left(\frac{d^4}{dt^4} + 2500 \frac{d^3}{dt^3} + 100,656 \frac{d^2}{dt^2} + 1000 \frac{d}{dt} + 2.6 \right) \beta$$

whereas the assumed factored form, when multiplied out, is

$$\left(\frac{d^4}{dt^4} + 2500 \frac{d^3}{dt^3} + 100,025 \frac{d^2}{dt^2} + 1000 \frac{d}{dt} + 2.5 \right) \beta$$

For the purpose of choosing parameter values, therefore, the factorization seems justified.

5. GRAVITATIONAL DETERMINATION OF THE VERTICAL

5.1. SELECTION OF METHOD

In section 3.3 the principle of operation has been expounded for a device to locate the direction of the vertical by means of the earth's gravitational field. It remains to comment upon the practicability of a physical realization of such a device, and upon its inherent errors and their design implications.

In effect, the physical problem is to measure relative to the vehicle the direction of a very small gravitational force. There are certain obvious ways of doing this by balancing the gravitational force with another force whose direction is known. However, many of these which may be appropriate to the laboratory (e.g., the balancing of a charged mass with an electric field against the gravitational field) seem not to be suitable for automatic mechanization. One which does seem feasible is a simple pendulum consisting of a mass suspended below the vehicle center of mass by a fine fiber. In the absence of angular accelerations, such a pendulum alines itself with the direction of the local field gradient. Its deviation from its normal position relative to the vehicle is directly the angular deviation of the vehicle zenith direction from the direction of the gravitational field, and is a measure of the pitch and roll error of the vehicle.

It may be noted that such a pendulum is just a special case of the Eötvös balance, which was designed to measure field gradients at the surface of the earth in the presence of an ambient field 5×10^6 times as great as that experienced in the satellite. An analogous device can be constructed by allowing a small sphere to roll in a spherical dish, rather than by suspending it by a fiber. However, accidental contamination of the surfaces may give rise to adhesive effects which make the instrument unreliable. Still another possibility is a double pendulum with damping between pendulums. However, unless the simple pendulum appears less favorable after further study, it is the recommended device of this class for the sensing function.

5.2. SYSTEM ERRORS

It is possible to consider some of the errors expected to affect this method of determining the vertical, at least qualitatively, without reference to the general equations of motion.

Errors of the Geoid

A stationary pendulum at the instantaneous position of the satellite will point, not toward the center of the earth, but normal to the local equipotential

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surface of the earth's gravitational field, i.e., normal to the geoid. Therefore, if the desired indicated direction is either geocentric or normal to the earth's surface, the pendulum may be expected to have a maximum angular error of 2 milliradians from this source.

Errors Due to Vehicle Angular Acceleration

A second class of errors arises from the angular accelerations of the vehicle about its center of mass. These may be examined from a rather naive point of view for the simple case of pitch motion alone. Suppose that a proof mass is mounted below the longitudinal axis of the vehicle at a position characterized by (a, λ). When the vehicle axis is inclined to the earth's tangent plane by an angle θ_2, as shown in Fig. 17, the forces on the proof mass are its apparent weight and the inertial reaction force arising from the angular acceleration $\ddot{\theta}_2$. It is easy to see that the inclination error is

$$\tan \delta = \frac{ma\ddot{\theta}_2 \sin(\lambda - \theta_2)}{[ma\ddot{\theta}_2 \cos(\lambda - \theta_2) + W_a]} \qquad (54)$$

Using $m = W_e/g$, and W_a from Eq. 5 with $\rho = a \sin\lambda$, the error angle is

$$\tan \delta \approx \frac{\ddot{\theta}_2 \sin(\lambda - \theta_2)}{[\ddot{\theta}_2 \cos(\lambda - \theta_2) + 3 \times 10^{-6} U \sin(\lambda - \theta_2)]} \qquad (55)$$

(Here U has value unity and dimensions $\sec^{-2}$.) The effect of this error depends upon the control system which operates on the information furnished by the apparent inclination. Without entering upon the details of possible systems of this kind, suppose that the control system responds to apparent inclination θ_2^* by applying a torque $-C\theta_2^*$ to the vehicle. To simplify the argument without loss of generality, suppose also that there are no time lags in the control system, that $\lambda = \pi/2$, and that $\theta_2 << \pi/2$. If the moment of inertia of the vehicle in pitch is J, then $J\ddot{\theta}_2 = -C(\theta_2 + \delta)$ and $\delta \approx \ddot{\theta}_2/3 \times 10^{-6} U$. The approximate equation of motion of the satellite in pitch is therefore

$$\ddot{\theta}_2 + \left[\frac{C}{(J + 10^6 C/3U)}\right] \theta_2 = 0 \qquad (56)$$

and the vehicle oscillates about the pitch axis with a frequency $\sqrt{\dfrac{C}{J + 10^6 C/3U}}$

and an amplitude determined by initial errors in the system. If there were no inertial reaction on the proof mass, the equation of motion would be identical except for a change in natural frequency. It should be noted that damping undoubtedly would be introduced in an actual mechanization of such a sensing and control system. It may be concluded that the error in inclination caused by inertial reaction is not necessarily serious, but that its effect depends upon the detailed nature of the attitude control system. If this system is suspended precisely from the center of mass, there is no error of this kind in any case.
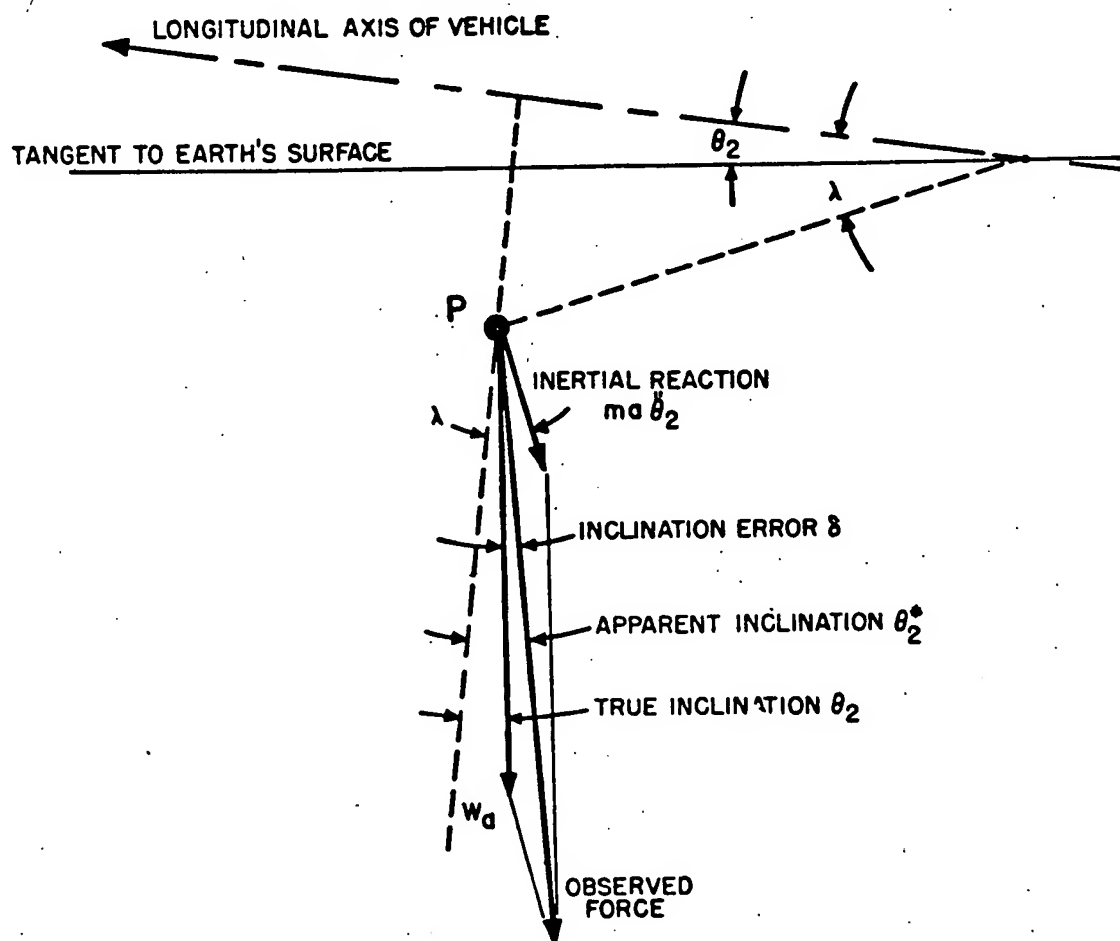
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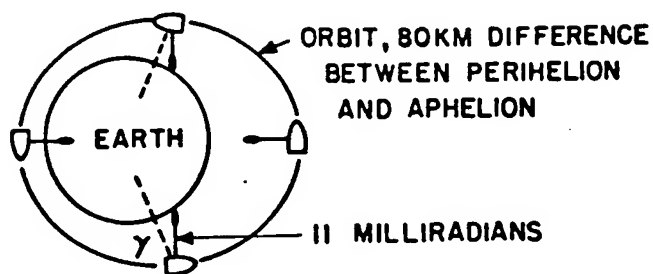Fig. 17.   Error Angle in Determining Vertical by Gravitational Means

Fig. 18.   Deviation of Plumb Bob for Elliptical Orbit

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Orbital Errors

If the satellite is rotating about its pitch axis at a uniform rate, with a period equal to the orbital period, and if the orbit is elliptical, the line from the center of the earth to the instantaneous position of the satellite is not normal to the satellite forward axis. The apparent effect is aggravated by Kepler's third law of planetary motion, which states that equal areas are swept out in equal lengths of time. Thus, certain quadrants of the ellipse are traversed in less than 1/4 orbital period, or 1/4 satellite pitch axis rotation. If the orbit is such that the maximum distance from the center of the earth is 80 km more than the minimum, there will be an apparent shift of the vertical of 11 milliradians, as shown in Fig. 18. With the pendulum bob rotating around the center of mass of the satellite at a constant rate of once per orbital revolution, the 11-milliradian component of gravity appears as a perturbing force on the pendulum.

Other Errors

Additional errors may arise from a variety of causes, being of more or less importance according to the care which is taken to reduce them. Among these are effects related to

1. Attraction of parts of the vehicle on the pendulum bob.
2. Thermal gradients in the neighborhood of the pendulum.
3. Electric and magnetic fields in the neighborhood of the pendulum.
4. Mechanical contamination or damage.
5. Reaction forces of measuring instruments.
6. Pendulum damping.

Mass attraction to the rest of the satellite will introduce an apparent acceleration which much be calculated for the final configuration. The magnitude of this component is calculated as 7.1×10^{-8} (m/sec)/sec acceleration for a 500-kg mass 1 m from the pendulum bob. The satellite design specification should include some tolerance on the amount of mass shift occurring during satellite operation.

It appears to be important that the bob, the suspension wire, and the walls of the container all have conducting surfaces in order to eliminate the possibility of electrostatic attraction. It is also important to keep parts at a uniform temperature to eliminate any thermoelectric potentials and associated forces if maximum sensitivity is required. Aluminum is probably not a satisfactory material because of its nonconducting oxide coating; however, copper, silver, or gold should be satisfactory. It is important to use a bob completely free of magnetic materials in order to avoid magnetic attraction to the damping magnets. The degree of purity used in sensitive galvanometer coils seems to be adequate. It is also important that the stops and caging mechanism be free from oil and make a small area contact in order to avoid danger of sticking from surface or intermolecular forces.

Some errors can be reduced ultimately by damping the pendulum, but the damping may introduce its own error. No errors will arise from its use if the damping can be inserted between the pendulum and inertial space. However, practically it must be inserted between the vehicle frame and the pendulum. Any frame accelerations will be communicated to the pendulum as spurious forces, resulting in errors. It will probably be desirable to cut such damping out of the system during periods of deliberate vehicle angular acceleration (i.e., periods of control).

5.3. DESIGN CONSIDERATIONS

Choice of Period

Disturbances from the ellipticity of earth and orbit occur at 1/2 the orbital period and at the orbital period. Hence, it seems desirable to avoid these periods in the selection of the proper pendulum period. A pendulum period large compared to the orbital period is attractive because of the natural attentuation of a low frequency system to higher frequency inputs. Subject to the limitations of avoiding the periods of exciting forces and of staying within the design limitations of the pendulum, there seem to be four choices, all with their own peculiar advantages and disadvantages:

1. A pendulum period of approximately 20 min (1/5 orbital period) with approximately 0.2 critical damping. This pendulum tends to follow the changes in the gravitational field and to point midway between the geographical and geocentric verticals. This ability to follow the gravitational field is of considerable value with an elliptical orbit because it allows the attitude control to point the camera toward the earth rather than toward the center of the elliptical orbit. This period pendulum also occupies less space in the satellite than the longer period pendulums.

2. A pendulum period intermediate between orbital and 1/2 orbital period: e.g., $1/\sqrt{2}$ orbital period. A pendulum of this period has close to the maximum possible ratio of gravitational forces to suspension bending restraints that can be obtained in a given diameter satellite. (A period of $\sqrt{2/3}$ orbital period puts the satellite center of mass in the center of the suspension and uses the longest possible suspension and greatest obtainable gravitational field.) This period pendulum may be expected to have the greatest stability of calibration. With approximately 0.05 critical damping, it oscillates with respect to inertial space at approximately twice the amplitude of the orbital and the twice-orbital period disturbances. Because of the high ratio of gravitational to suspension bending forces (approximately 300 to 1), the motion of the pendulum when once determined by observations over known territory and extrapolated to a complete orbit should be predictable. The small amount of damping is added to damp out the initial transient, and any stray disturbances arising from satellite control or other causes. With 0.05 critical damping, a disturbance will damp to within 37 percent of its initial value in 3 cycles. Near-

critical damping would be expected to be a disadvantage in a high accuracy vertical indicator because of the coupling between the satellite motion and the pendulum.

3. A pendulum period of approximately $\sqrt{2}$ orbital period. Roughly, the same considerations apply to this pendulum as to the pendulum of the previous paragraph, except that the ratio of gravitation restraint to bending restraint is not as good by a factor of 4 and that it is less stable. It has the advantage of attenuating the twice orbital period motion by a factor of seven. It can also have considerable advantage when properly combined with a pendulum whose period is shorter than orbital period, say $1/\sqrt{2}$ orbital period. This is because of a nearly 180-deg phase shift between the response of the shorter and longer period pendulums to orbital period disturbances, particularly if the damping is 0.01 critical or lower (0.01 critical requires 16 periods to damp an initial transient to 37 percent of its initial value). By choosing appropriate ratios of areas of capacitor pick-off and paralleling the pick-off for $1/\sqrt{2}$ and $\sqrt{2}$ orbital period pendulums, it is possible to have a mean position which either (1) points very closely to the center of the orbital ellipse and thus requires a minimum of control action; or (2) points very closely to the direction of the acceleration gradient and thus requires the least correction of photographic data for orbital ellipticity. With this system and 0.05 critically damped pendulums, attenuation of orbital period disturbance can be 4 to 1 or 18 to 1 if the damping is reduced to 0.01 critical.

4. A pendulum in a seismic suspension where the period is 4 or 5 orbital periods. This pendulum tends to average out all perturbations and to point to the center of an elliptical orbit. The five times orbital period pendulum with 0.05 critical damping attenuates orbital period disturbances by a factor of 25 and calls for the minimum amount of satellite attitude control action. Unfortunately, it is so critical to initial alinement that a shorter period pendulum may be required until the uncaging transients are damped.

The arrangements and design limitations of such pendulums depend in large measure on the vehicle configuration. The minimum period pendulum that can be designed in a 6-ft dia satellite is of the order of 20 to 25 min. A 22-min period pendulum (Fig. 19A) with a 0.5-kg bob, requires 0.5 mil or smaller quartz fiber to keep the fiber restraint to less than 20 percent of the gravitational restoring force. A 13×10^{-6} m (13 micron) dia quartz fiber has a breaking strength of 0.3 newton and requires caging the bob during ground handling. The maximum period simple pendulum that can be satisfactorily designed appears to be of the order of 2 times the orbital period, and is arranged as in Fig. 19B.

A larger period pendulum has the bob so close to the satellite center of mass as to reduce seriously the gravitational restoring force. The requirement that the component of the acceleration of the point of attachment along the direction of the string be kept less than the net gravitational field at the bob

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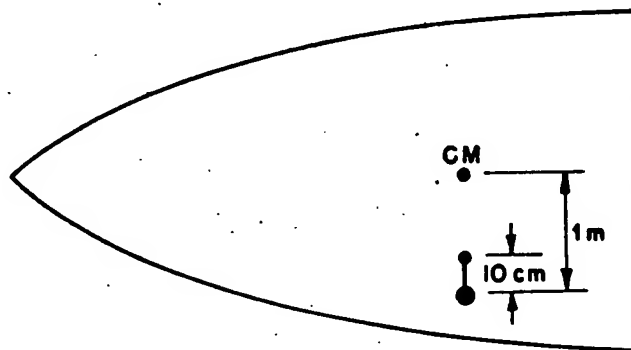
A. Arrangement of a 22-Min Period Pendulum
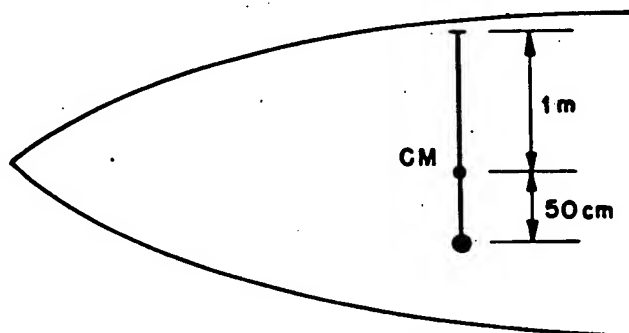
B. Arrangement of a $1/\sqrt{2}$ Times Orbital Period Pendulum
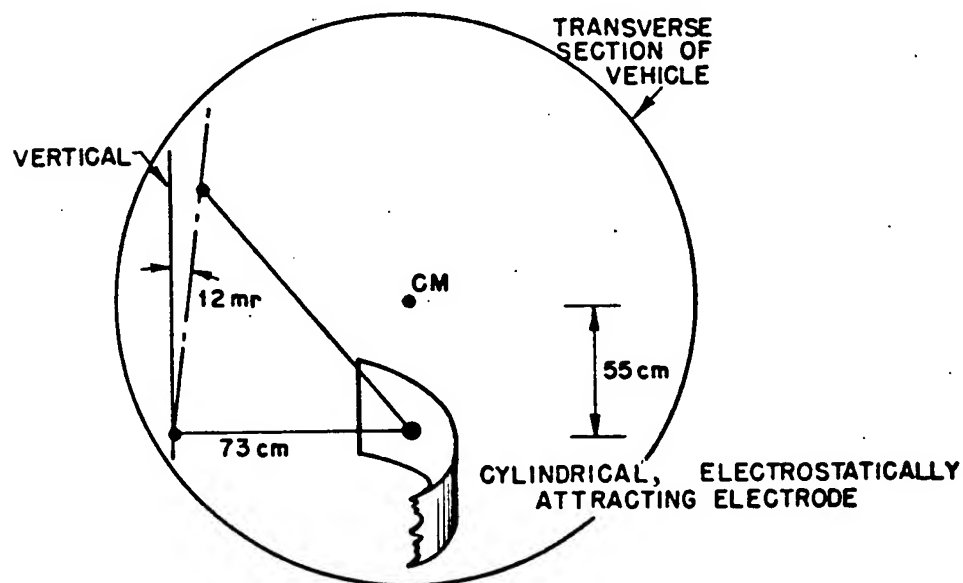
C. Arrangement of a 5 Times Orbital Period Pendulum

Fig. 19. Pendulum Arrangements for Determining Vertical

limits the permissible maximum control acceleration and velocity. This limitation on maximum period is changed, of course, if a vehicle configuration is used which permits longer suspension lengths. A pendulum of this type with, say, 5 times orbital period has the center of mass of the bob within about 1 cm of the satellite center of mass, so that its period is unduly influenced by the mass of residual fuel or other mass shifts.

A pendulum whose period is 5 times orbital period (which attenuates orbital period disturbances by a factor of 25) can be patterned after a seismic pendulum (Fig. 19). This design is more delicate than that of a $1/\sqrt{2}$ orbital period pendulum and requires separate pendulums for pitch and roll. It requires better initial alinement: pitch, for instance, becoming completely unstable if roll is more than 12 milliradians misalined with the gravitational vector. It is also necessary to use approximately 0.5-mil quartz fiber or smaller with a 0.5-kg bob in order to reduce bending restraint to about 5 percent of gravitational forces. The electrostatic forces necessary to hold the suspension taut can be supplied by 100 v across a 3-mm gap with a 12-sq cm area on the bob facing the cylindrical electrode.

Pick-Off Design

A capacitor type pick-off is recommended because of its low circuit power requirement, its low, easily-calculated coercion, and its simplicity and inherent reliability as compared to a photoelectric pick-off and light source. If the bob is a 25-cm hollow sphere, and the pendulum has a $1/\sqrt{2}$ orbital period with ±17-milliradian angular freedom, then the pick-off capacitance can be in the order of 10 micromicrofarads. The maximum allowable voltage across the capacitor plates is 0.2 v rms if electrostatic forces, with the bob displaced to a stop within 10 percent of the displacement to the capacitor plates, are not to exceed 10 percent of the bob restoring forces. The sensitivity of the pick-off should be limited only by circuit noise and physical stability of circuit parts. If a 1-mc carrier (presumably a higher frequency already used in the satellite is available) and a 5-kc bandwidth are used, the thermal agitation noise in a resistive circuit whose impedance is equivalent to 10 micromicrofarads is 1 microvolt. Hence, the limitation is circuit physical stability rather than noise, and it should be possible to detect approximately 2×10^{-5} radian pendulum motion with circuits of home radio receiver quality, and much finer motion with improved circuits. It may be feasible to use two carrier frequencies and have one pendulum serve both roll and pitch.

Suspension

Quartz was chosen as the suspension material for this discussion primarily because suitable techniques for drawing it straight have been developed by others, and because improper handling will break it before introducing an

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unknown permanent bend. The quartz fiber will, of course, require an evaporated metallic coating to make it conducting and to eliminate surface electrostatic charges. Tungsten is about twice as strong as quartz in the sizes considered, but the uncertainty in the straightness of the fiber and the five times higher modulus of elasticity for tungsten are considered as overruling disadvantages. Suspensions have been considered as 1-mil fuzed quartz unless otherwise stated.

For a given ratio of suspension strength to bob weight, reducing the bob weight by a factor of 2 reduces the suspension diameter by the $\sqrt{2}$ and the section modulus by 4. Hence, the ratio of spring restraint to bob weight is reduced by a factor of 2 by reducing bob weight by a factor of 2. However, if carried too far, the suspension becomes so small as to be unreasonably difficult to handle.

The absolute accuracy of the pendulum in the absence of perturbations seems to be determined by the magnitude of residual bends in the suspension fiber. Bends in the suspension can be observed on the ground by making the suspension with integral tabs on each end, and by observing how straight the suspension hangs from its own weight. The tabs could be used to prevent misalinement during bob attachment.

The stability of the pendulum null position appears to be determined by the stability of the initial bending strains in the suspension, which change with temperature because of the temperature coefficient of the modulus of elasticity of quartz. This is 2.3×10^{-4}/deg F for quartz fibers, and if temperature changes are held to 50 deg F, there will be a 0.02-milliradian change in zero position for a pendulum with 2-milliradian initial bias. In this calculation, it is presumed that the metallic coating makes a negligible contribution to the elastic constants of the suspension. The major remaining cause of zero drifts is expected to be circuit instability, caused primarily by dimensional changes in the coils associated with the pick-off. Proper design should keep these effects to less than 0.1 percent of the pick-off range.

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6. DETERMINATION OF THE VERTICAL BY HORIZON SCANNING

6.1. SELECTION OF METHOD

Background

Considerable work has been done on horizon detection for sea-level and aircraft use. Some of this (Ref. 16) shows that the horizon can easily be detected in clear weather, but that a distant cloud gives very similar signals. Near clouds, of course, may obscure the horizon completely. However, the same problems should not exist for a satellite.

The devices which have been reported for horizon sensing in general are large, require some human intelligence to differentiate between clouds and horizon, and have a limited angle through which they will servo toward the horizon. Moreover, for applications of navigation, they were designed to have an accuracy somewhat greater than necessary for the satellite sensing problem.

Ample radiation (although not its exact value) is known to exist at the satellite altitude to make a horizon sensing system of a similar kind operable. The problem is the development of a sensing system of minimum weight and power consumption and high resolution which will give correcting signals over a wide angle of attitude deviation.

There is assumed, of course, a system sensitive enough to satisfy the operational requirements on it, although its development may be no trivial problem. The feasibility of such a system can be considered to have been demonstrated when a physical device (operating on the principles discussed below) is constructed which locates the horizon within the order of 5 milliradians under field and operational conditions.

Daylight vs Continuous Operation

The kind of horizon scanning system which might be used for sensing the vertical is strongly conditioned by its operating requirements. One of the first questions to be considered is whether operation can be restricted to daylight, or must be continued at night.

A liberal estimate of the expected rms perturbation torque is 10^{-4} newton-meter. If a conventional configuration is used for the vehicle, and a horizon scanner is permitted to cease operation during the satellite night of 50 min, then a continuously applied torque of this magnitude results in the accumulation of an angular error of nearly 0.5 radian during the unsupervised period. On the other hand, if the same conditions maintain for a vehicle in the stable

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attitude, a maximum angular error of about 0.1 radian occurs during the un-supervised period. This appears large, but if there has been an overestimation of the perturbation torque by a factor of ten, the actual expected attitude error probably will be tolerable. Thus, it does not seem possible at this time to give a definite answer as to whether a daylight-only system is sufficient.

In general, the techniques for the two cases will be similar. However, at least two factors will reduce over-all sensitivity at night. First is the fact that although during daylight the radiation will come primarily from reflected and scattered sunlight, which is quite strong, at night the only radiation will be the thermal radiation from the ground and atmosphere. Second is the fact that thermal detectors, which have a lower sensitivity than visible light de-tectors, will have to be used for night operation. The combination of lower radiation intensity and lower detector sensitivity necessitates the use of larger optical systems and higher gain amplifiers for night operation.

A Supplemented Daylight System

There are alternatives to continuous operation or operation in daylight alone. One such possibility is to maintain the vertical by means of a vertical gyro during the time the horizon scanning is not in operation. This provides a vertical reference both during the night and during any periods when the sun appears in the field of view of the scanner, obscuring the horizon. The oper-ational requirements on the gyro from the point of view of drift rate and gyro life, are no more stringent than on the yaw gyro. Therefore, if the design feasibility of the latter is demonstrated, it will be assured that a vertical gyro can be constructed.

A gyro vertical might operate in the following way. During the day, when the horizon sensing system operates and is in control, the gyro is torqued so that its resultant precession keeps its axis alined with the local vertical (as determined by the horizon-seeker). During the night, the same average pitch rate is imposed on the gyro by a suitable constant torque, and the vehicle should have an attitude close to its desired value when the horizon sensing sys-tem is again ready for operation.

However, a probably more desirable system is one in which the gyro controls the attitude at all times. The horizon scanning system is considered a long period monitoring system (when it receives radiation between two pre-set intensity levels), which supplies torque changes to the gyro torquers in order to prevent a drift of the gyro away from its desired orientation. Such a system is simpler than one in which vehicle control alternates between two dif-ferent sensing systems. Seen in this light, the supplemented gyro system re-sembles in some respects a stellar-supervised inertial autonavigator system, but is considerably less complex.

It seems that a vertical gyro with a daylight horizon scanning monitor is entirely feasible and will result in a continuous satisfaction of accuracy requirements. However, further studies are desirable on the details of such a system.

Other Design Considerations

For night use, the system must operate through one or more windows of rock salt, arsenic trisulfide or other infrared transmitting material. These windows need covers for protection in passing through the atmosphere because of the adverse effects of heat and moisture on their optical properties, but these covers could be removed for use by an explosive charge operating from a vacuum-sensitive detonator. For a daylight-only system, the windows may be of pyrex or quartz, which should need no protection.

For convenience in configuration design, it is felt that four receivers viewing the fore, aft and beam directions through windows in the sides of the vehicle are preferable to a bubble or large plane window in its belly.

It may be possible to reduce the number of receivers by using one or two rotating scans, comparing in two receivers the angular dip seen in two directions π radians apart. With some complication in the optical arrangements, and a corresponding change in receiver circuit, it appears that a feasible system which uses only one receiver can be designed for daylight operation. However, the system discussed in this report uses two receivers oppositely directed. Such a system gives an attitude error measured in the plane containing the receiver axes.

Two possibilities exist for mounting the receivers. They may be either in mounts fixed relative to the vehicle frame or they may be gimballed. The mechanical and electronic complications of gimballed receivers which are required to servo to the horizon are sufficiently great to urge the use of fixed receivers if possible. There seem to be no serious objections to the use of fixed receivers provided that their total field of scan is sufficiently large that the horizon will not be lost even when the attitude error of the vehicle is several degrees. This provision does, however, limit the ultimate resolution of the receivers.

6.2. COMPONENTS OF HORIZON-SCANNING SYSTEM

Sensitive Elements

In general, there are two types of infrared sensitive elements: total thermal radiation receivers and quantum receivers.

Representative types of total radiation receivers are thermocouples, metallic bolometers, thermistors and Golay pneumatic cells. With the exception

of the Golay cell, these usually have low impedance and long time constants (about 0.1 sec), although some have been made with a time constant as short as 1 millisecond. Figure 20 shows comparable cell sensitivities as obtained several years ago, but more recent information suggests that most of these cells have been improved by a factor of about ten.

The quantum receivers are represented by photoemissive, photovoltaic and photosensitive semiconductors. Neither photoemissive nor photovoltaic cells are sensitive very far into the infrared. The photosensitive semiconductors (commonly called photoconductors) have considerably higher sensitivities but are limited to the region from the visible to about 3 to 6 microns. The exact upper limit depends upon the cell material and its processing. At present, the best materials known are lead sulphide, lead selenide and lead telluride. The last, it should be noted, requires liquid air cooling. Cells of such materials have high impedances and are capable of responding to signals of higher frequency than can be detected by the total radiation receivers. Lead telluride cells, in particular, have been built which respond to 1-microsec signals. However, extremely short response times are not a question in the satellite application.

Selection of the Cells

A number of considerations influence the choice of a cell for the present application, this choice being one of the major problems in the design of an horizon-scanning system.

There is some question about the best spectral region to be used. With daylight operation only, the spectral region in which the atmosphere scatters sunlight most strongly (the near ultraviolet) may be preferable. The earth's water vapor sphere, which radiates in the near infrared, may be usable for combined day and night operation. With long wave infrared detectors, ground radiation may be used, but the earth is not always detectable through clouds and water vapor. Tests at ground level and some analytical considerations show that the lead sulfide cells may not be able to detect a night horizon.

Tousey and Hulburt (Ref. 17) have calculated and measured the light scattered by the atmosphere at various altitudes. Packer and Loch (Ref. 18) have also measured this light at altitudes between 18,000 and 38,000 ft and have obtained agreements with the Tousey-Hulburt values. The theories indicate that the radiation from the sun in passing through the atmosphere is scattered equally in all directions, so that the brightness in a particular direction of view is primarily a function of the length of path in this direction through the atmosphere. Consequently, sky brightness measurements of the sky near the horizon made at various altitudes in clear weather show nearly the same results because the direction of view passes through the same distance of atmosphere.

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A. Horizon Scanner

CLOCK MOTOR
MIRROR
SHUTTER
SOLENOID
INFRARED CELL
PROTECTING VANE
PHOTOVOLTAIC CELL

B. Scanning Pattern

SPACE
EARTH
RECEIVED RAY BUNDLE
SCANNER
1°
6°
BAND WITHIN WHICH HORIZON MAY APPEAR

Fig. 21. Infrared Horizon Scanner and Scanning Pattern

THALLOUS SULFIDE
LEAD SULFIDE
CAESIUM SILVER OXIDE PHOTOEMISSIVE CELL
SUPERCONDUCTING BOLOMETER
LEAD SELENIDE
GOLAY PNEUMATIC CELL
LEAD TELLURIDE
THERMISTOR
BAIRD BOLOMETER
POLAROID BOLOMETER
THERMOPILE

RADIATION ON CELL FOR UNIT SIGNAL-NOISE RATIO, WATTS

WAVE LENGTH OF RADIATION, MICRONS

10^{-11}
10^{-10}
10^{-9}
10^{-8}

0.5   1   2   4   8   16

Fig. 20. Comparison of Cell Sensitivities

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From the height of the satellite, the minimum radiation at the horizon will occur when the weather is clearest. This is shown by the following reflectivities of the earth's surface constituents (Ref. 17): clouds, 50 to 80 percent; desert, 20 to 25 percent; forest, 4 to 10 percent; sea, 3 to 7 percent.

Consider the minimum radiation case as that of clear sky, above which there is a brightness of approximately 10^4 candle/sq m (Ref. 18). The flux density in lumen/sq m at a point from an infinite source is given in Ref. 19 as the product of source brightness b (candles/sq m) and the solid angle of view of the receiver (steradians). For an optical system of focal length f and receiving cell diameter d, the solid angle of view is $(d/f)^2 \pi/4$ steradians.

The total flux on the cell is a product of collector area and flux density at the collector, or $(Dd/f)^2 b\pi^2 16$, where D is the diameter of the optical system. For $D = 5$ cm, $f = 10$ cm, and $d = 3$ mm, there is 0.014 lumen of flux. Information on the sky brightness spectrum does not seem to be available, so that cell sensitivity information cannot be applied directly. To make some estimate of cell response, suppose that a PbS cell has a spectral curve similar to that of the human eye and a sensitivity of 10^{-1} v/microwatt (as given in Ref. 20 for a commercial cell). The wider actual spectral response of photoconductor cells will result in a greater than calculated output. For radiation at peak eye sensitivity, the correspondence 625 lumen/w may be used, so that the equivalent cell sensitivity is about 160 v/lumen. The signal from this cell with a flux of 0.14 lumen is about 2 v. Because this signal is further amplified, it is ample for satisfactory operation.

The cell choice affects the circuit components which can be used in the receiver. A study of magnetic amplifiers indicates that these may be used with advantage for low impedance infrared cells. However, a vacuum tube preamplifier is required for high impedance cells, such as might be used with visible or ultraviolet radiation.

Details of an existing receiver which uses a 2-in collector, a PbS cell detector, and a 1-kc tuned amplifier are given in Ref. 21. This system gives a strong daytime signal at the horizon, but has not yet shown a strong signal at the horizon during the night.

6.3. DESIGN CONSIDERATIONS

Principle of Operation

The scanner shown in Fig. 21A can be used. As the scanner mirror rotates, the sensitive cell sees the swept-out area shown in Fig. 21B. When the vehicle is within a set limit, say ± 3 deg, of the proper attitude, an error signal is generated by the relative amounts of scan above and below the horizon. This

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results in a signal roughly proportional to the attitude error. By adjusting the cone swept out by the scanner, this permitted angular deviation is made larger or smaller as desired. When the attitude error is so great that one receiver scans only earth and the other only sky, the output remains constant at the maximum error value as long as this condition exists.

The primary elements of this receiver are a rotating concave mirror, a shutter, a sensitive cell, an amplifier, and a comparison circuit. A clock type electric motor rotates the mirror, the optical axis of which is at an angle to the axis of rotation so that the sensitive cell scans the field of view in a circular path. The shutter located in front of the mirror is composed of a fixed sector disk and an identical sector disk fastened to the mirror. As the mirror rotates, the opaque sectors of the rotating disk alternately fall on the open and opaque sectors in the fixed disk. This results in a modulation of the radiation focused upon the cell by the mirror.

When the receiver on one side scans only the sky and that on the other side scans only the ground, a difference in signal amplitude will exist in these two receivers. The signal from the cell in each receiver is amplified to the necessary signal level, rectified, and filtered. As shown in Fig. 22, the signals from two diametrically opposite receivers are rectified in the opposite polarity so that the sum of these two signals will be plus or minus depending upon which is the larger. This sum signal will be the coarse servo signal, for it will indicate which receiver is scanning sky and which is scanning ground. It will indicate which way the vehicle should turn to line up with the horizon, although it will not show how great the error is.

When the receivers scan across the horizon, signals of alternating amplitude are generated as shown in Fig. 22C. These signals are rectified and go through a set of filters which have a time constant short enough to obtain the envelope of the modulation which represents the relative portions of the scan circle above and below the horizon. These envelope signals are shown at c_1 and c_2, and after amplifying and limiting at d_1 and d_2. The signals are π radians out of phase because the first rectifiers are reversed from each other. The envelope voltages are rectified, filtered, and added, their sum being a measure of the amount one receiver of the pair scans below the horizon more than the other. The signal will be independent of altitude of the vehicle, within limits, because the same change in the height of the horizon on the scanners in a pair will cause identical changes in the lengths of the pulses representing sky scan. These changes are positive in one recei.. _r and negative in the other, so that the signals caused by the change will cancel. Identical systems would be used at right angles to control the satellite in a second attitude.

To prevent the sensitive cell from being burned out when the receiver looks directly at a rising or setting sun a protective vane may be caused to cover it by the energy generated in a photovoltaic cell.

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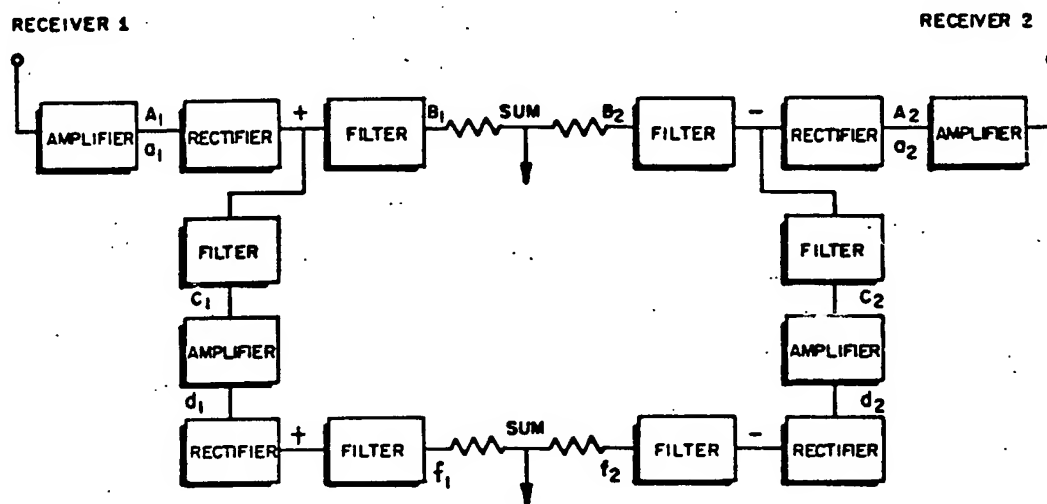
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A. Block Diagram of System

B. System in Bang-Bang Operation

C. System in Proportional Operation

Fig. 22. Receiver Signals for Horizon Orientation

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Errors

In setting the performance characteristics of the horizon-sensing system, it is important to know something of the changes in the dip of the horizon in the various directions as seen from a single point on the o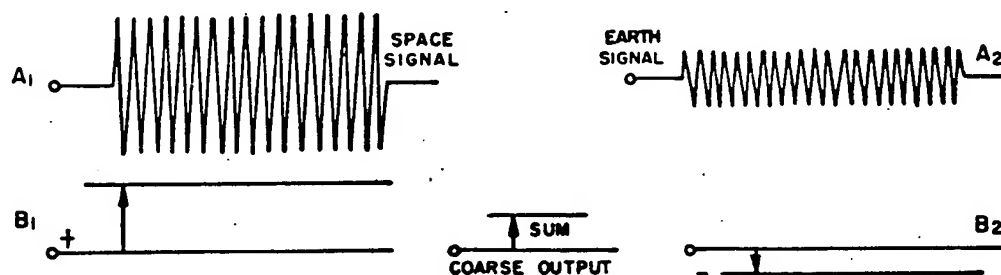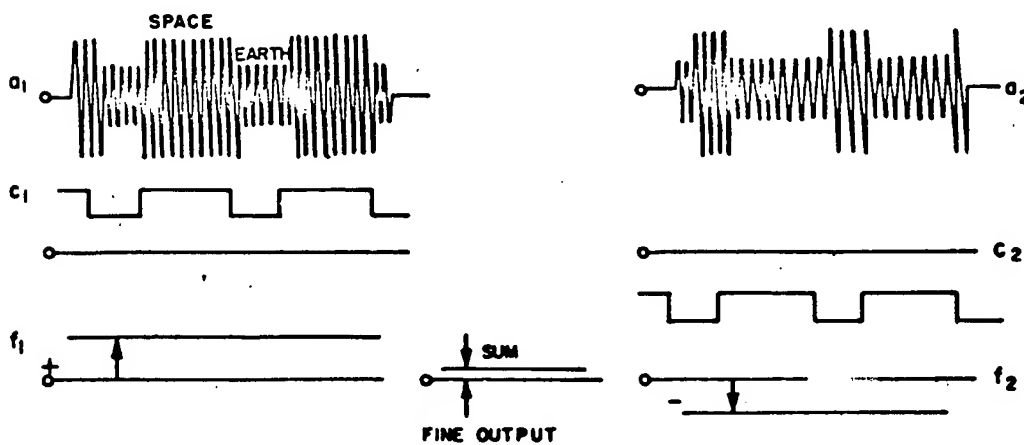rbit, as the vehicle progresses around its orbit. In particular, the ultimate accuracy of the instrument is limited by the difference in dip in the two directions of each line-of-sight along which its axis may be directed. The fundamental errors are caused by the ellipticity of the earth, by topographic differences in the two directions, by differences in cloud or water vapor levels to which the device may be sensitive, and by orbital ellipticity.

No error analysis considering these factors in detail has been completed. However, some estimates of the difference in dip angle caused by several effects can be made by a simple argument. Let R be orbital radius and R_h the radius of the earth at the horizon, as seen from the vehicle. Let ϵ be the dip of the horizon. Because $R_h = R \cos \epsilon$, the dip change caused by changes in R_h (i.e., by the earth's ellipticity, by clouds or by mountains) is $|\Delta\epsilon| = |\Delta R_h|/R \sin \epsilon$; and that caused by changes in orbital radius is $|\Delta\epsilon| = |\Delta R_h| \cos \epsilon /R \sin \epsilon$.

As a numerical example, it may be assumed that average $R = 6900 \ km$, $\epsilon = 0.425$ radian. If $|\Delta R_h| = 35 \ km$, and $|\Delta R| = 80 \ km$, the dip changes are respectively $|\Delta\epsilon| = 12.5$ milliradians and $|\Delta\epsilon| = 26.1$ milliradians. However, it must be remembered that these angular dip changes are not necessarily errors, as a similar change may be seen by the oppositely directed receiver. In particular, the second change above may be expected to be the same in both receivers, and give no error signal. (It is important to know the value, though, as the field of scan must be chosen large enough to encompass such expected changes.) The full value of the first change above, due to ellipticity, mountains and clouds, may be expected to appear at times as an error.

7. METHODS OF CONTROL

7.1. CLASSIFICATION OF METHODS

One must choose a control system consistent with the nature and magnitudes of the perturbation torques against which it is to act in maintaining the desired attitude. It has been shown in Chapter 2 that the perturbation torques are generally small, but that they depend critically on the final configuration of the satellite, and upon the nature of its moving parts. Therefore, a quantitative answer cannot yet be given to the question: "What are the nature and optimum design parameters of the best attitude control system for the satellite?" One can, however, discuss in a general way the kinds of control systems which might be conceived, their limitations, advantages and disadvantages.

Two exhaustive classes of control methods are apparent. A torque is to be exerted on the vehicle to cause angular accelerations and correct the attitude, which must be provided by a force or torque interaction between the vehicle and inertial space, or between the vehicle and its ambient field. In general, almost any effect which can cause perturbation torques can be used to obtain control torques. The methods which are considered in the sequel employ

1. Reaction gyroscopes
2. Acceleration wheels
3. Gyroscopic action of rotating parts
4. Changing moment of inertia
5. Radiation from the vehicle
6. Jet reaction thrust
7. Gravitational field of earth
8. Magnetic field of earth
9. Atmospheric pressure field
10. Radiation incident on the vehicle

7.2. CONTROL BY REACTION WITH INERTIAL SPACE

Reaction Gyroscopes

A method of control proposed in 1947 (Ref. 22) was based on the properties of gyroscopes. Consider a gyro mounted in a frame whose plane is constrained to remain in a vehicle-fixed plane. The frame, however, can rotate about an axis normal to the fixed plane. The control action of this gyro can be described with reference to the typical case shown in Fig. 23. If control motion about the z-axis is desired, the gyro spin axis lying along the x-axis,

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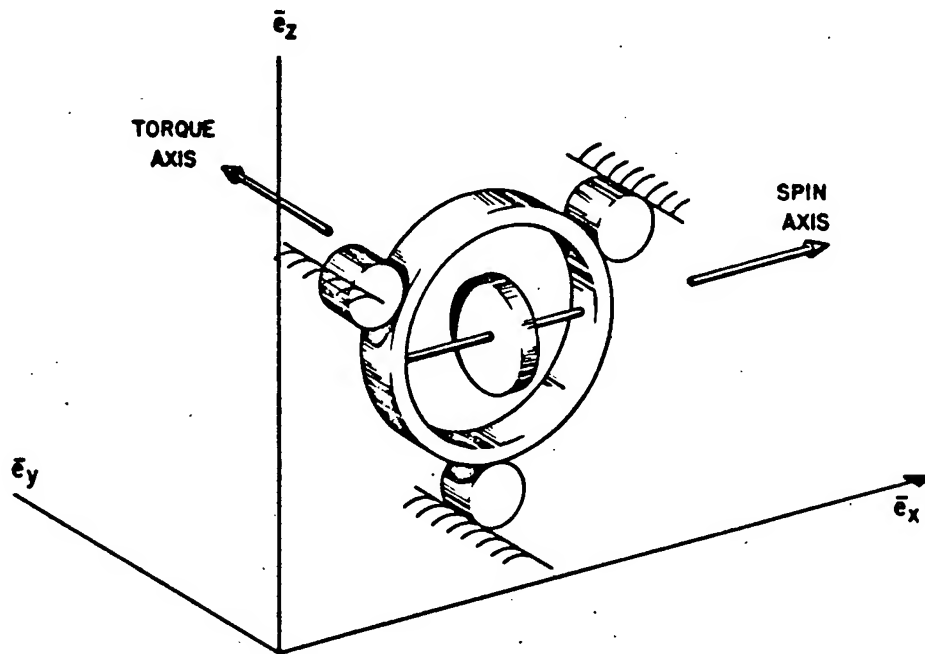
Fig. 23. Typical Reaction Gyro

a torque is applied between the vehicle-fixed xyz-axes and the gyro frame, the torque axis being the y-axis. The gyro then tends to precess about the z-axis; but, as the frame is constrained to lie in the xz-plane, the gyro precession must carry the whole vehicle with it about the z-axis. In simplest terms, this is the way in which a reaction gyro functions.

Difficulties are quickly encountered with this system, however. As the frame turns, the gyro axis becomes more nearly parallel with the z-axis, and as it does so, the precession component about the z-axis is reduced at the expense of an increasing component along the x-axis.

In Ref. 22 it was proposed to overcome this difficulty by using a frame of four gyros which would balance out unwanted precession velocities, and to return the system to its initial configuration after each half-revolution of the frame by a process of "flipping" the gyros. When the complete cylic process is used however, the principle of operation is fallacious.

The details of the proposed system need not be considered in order to show that this is so. Suppose that some mechanism causes changes in the direction, but not the magnitude, of the total angular momentum $\bar{H}$ of the system by the application of certain torques by the vehicle frame. If the reaction torque of the control system on the vehicle is $\bar{L}$, then the total impulse imparted to the vehicle by the control system during a time t is

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$$\int_0^t \bar{L} dt = \int_0^t (-\dot{\bar{H}}) dt = \bar{H}(o) - \bar{H}(t) \qquad (57)$$

If at time t the configuration of the system is made identical with its initial configuration (which is the purpose of the flipping process), the difference $\bar{H}(o) - \bar{H}(t)$ vanishes. In the system as proposed, over one cycle of operation, there is no net control torque impulse imparted to the vehicle, whatever may be the structure of the reaction gyro system.

Moreover, in any portion of an operating cycle, the maximum impulse magnitude which can possibly be obtained is $2|\bar{H}|$, which occurs when $\bar{H}(t) = -\bar{H}(o)$. If $|\bar{H}|$ is made sufficiently large to meet any needs expected under the operational requirements, such a system may be made feasible. However, a system in which $|\bar{H}|$ changes can be made simpler than that in which merely the direction changes. Thus, one is led to consider a system of acceleration wheels.

Acceleration Wheels

A system of control based on the use of acceleration wheels, or "flywheels," has been proposed in Ref. 1. Such a system has a much simpler structure than that of the reaction gyros, consisting merely of three flywheels whose axes are rigidly constrained along the three principal axes of the vehicle and a means for accelerating these wheels about their respective axes: i.e., torques applied by the vehicle structure to the wheels. The reaction torques of the accelerating wheels act on the vehicle frame and provide the desired control torques. The control torque impulses realized with such a system are limited only by the maximum angular momentum which can be stored in each wheel.

This method is of considerable interest, being apparently feasible, relatively simple, and able to provide a fine damped control to the vehicle. It is discussed in greater detail in **Chapter 8.**

Gyroscopic Action of Rotating Parts

It has been suggested that it is possible to build into the vehicle a certain inherent yaw stability by appropriate arrangement of the rotating parts within it (e.g., motors and generators) and by taking advantage of the fact that a stabilized vertical implies a non-zero average pitch angular velocity of the vehicle.

Consider the simple case of uniform angular pitch velocity ω_2. The variables are defined as in Chapter 4, except that the gyro is replaced by rotating parts, and these parts constrained to rotate about the vehicle pitch axis with angular velocity Ω. The principal moments of inertia of the combined system

of vehicle and rotating parts are I, J, K about the roll, pitch and yaw axes respectively. If the roll angle θ_3 and yaw angle θ_1 are small, there results a special case of the situation treated in Chapter 4. The yaw and roll behaviors of the vehicle are described by

$$K\ddot{\theta}_1 + \Omega J_G \dot{\theta}_3 + J_G \Omega \omega_2 \theta_1 = L_z \tag{58}$$

$$I\ddot{\theta}_3 - \Omega J_G \dot{\theta}_1 + J_G \Omega \omega_2 \theta_3 = L_x \tag{59}$$

The yaw behavior, therefore, is given by

$$IK\ddddot{\theta}_1 + J_G\Omega[(I+K)\omega_2 + J_G\Omega]\ddot{\theta}_1 + J_G^2\Omega^2\omega_2^2\theta_1 = I\ddot{L}_z + J_G\Omega\omega_2 L_z - \Omega J_G L_x \tag{60}$$

It can be shown that the solution of the homogeneous equation corresponding to Eq. 60 is purely oscillatory, with two natural frequencies which can be approximated by

$$\left\{ \frac{J_G\Omega[(I+K)\omega_2 + J_G\Omega]}{IK} \right\}^{1/2} \quad \text{and} \quad \left\{ \frac{J_G\Omega\omega_2^2}{[(I+K)\omega_2 + J_G\Omega]} \right\}^{1/2}$$

The static error in θ_1 for a constant torque about the yaw axis is

$$\theta_1 = L_z / J_G\Omega\omega_2 \tag{61}$$

Similarly, there is a static roll error due to any constant L_x of

$$\theta_3 = L_x / J_G\Omega\omega_2 \tag{62}$$

Typical numerical values of these static errors, may be $J_G \approx 10^{-3}$ kg-m^2, $\Omega = 1800$ rpm, $\omega_2 \approx 10^{-3}$ radian/sec. Then, if a typical value of L is 10^{-5} newton-meter (100 dyne-cm), the corresponding static error is about 5 milliradians. A value of J_G somewhat larger may be more realistic, and if so, the static error is reduced.

Persistent yaw and roll torques this large are not likely to occur. However, one of the natural frequencies of Eq. 60 is not far removed from the orbital frequency ω_2. Therefore, it is possible that cyclic processes with this frequency (e.g., changes in orbital curvature) will excite the roll and yaw modes near resonance, building up a significant error in these variables. Another difficulty with the system is that it does not damp out any yaw or roll

oscillations. Therefore, although this may be a valuable supplemental control system, its use alone for yaw control should be considered only with the reservations of a more complete analysis.

It seems clear, though, that unless the resonance effects mentioned are very severe, this method of at least partial control will necessarily be used. There are certain rotating parts within the vehicle which must be given some orientation. If it is possible to provide counterrotating parts in each instance, there is no interaction with the uniform pitch velocity no matter what their orientation. However, if this is not possible, it is probably much better to aline their angular momentum with the pitch axis than with either the roll or yaw axes. If this is done, some roll and yaw control torques perforce arise.

Changing Moment of Inertia

A discussion of inertial methods of control is not complete without including changing moments of inertia, exemplified by the classic example of a falling cat righting itself.

The principle of the method is illustrated by Fig. 24. The vehicle is shown schematically as two bodies which can undergo relative rotation about an axis $\bar{e}_x$. Each portion can have its moment of inertia about $\bar{e}_x$ changed from I_0 to I_1, by the extension of bosses. If it is desired to move the point A through an angle θ_3 to position B, and A' to B', shown in Fig. 24A, the following sequence of operations can be used:

1. With the bosses C' extended and C retracted, a relative rotation $(I_1 + I_0)\theta_3 /(I_1 - I_0)$ is imposed, so that A and A' take the positions shown in Fig. 24B.
2. C' is retracted and C extended leaving A and A' in the same positions, as shown in Fig. 24C.
3. The bodies are counterrotated until the line AA' is again parallel to $\bar{e}_x$, as shown in Fig. 24D.

In this way, a net rotation of both bodies occurs about $-\bar{e}_x$ through an angle θ_3. It is easy to show that the two relative rotations of the bodies during this sequence are of the same magnitude and opposite sign. Thus, given the moments of inertia and desired angle of net rotation, it is simple to mechanize the commands for a control sequence.

The disadvantages of the method are probably sufficiently important that it cannot be used in general. The most obvious disadvantage is that an exceedingly complex system is required to obtain control about more than one axis. Additional difficulties are: (1) the angular error θ_3 must be made even greater, probably intolerably great, before it can be brought to zero; and (2) because the

A. Initial Configuration: *C* Retracted, *C'* Extended

B. Left Body Turned Through $I_1\theta_3/(I_1 - I_0)$ *C* Retracted, *C'* Extended

C. As in B; but *C* Extended, *C'* Retracted

D. Left Body Turned Back Until *AA'* is Parallel to $\bar{e}_x$. Net Rotation is θ_3.
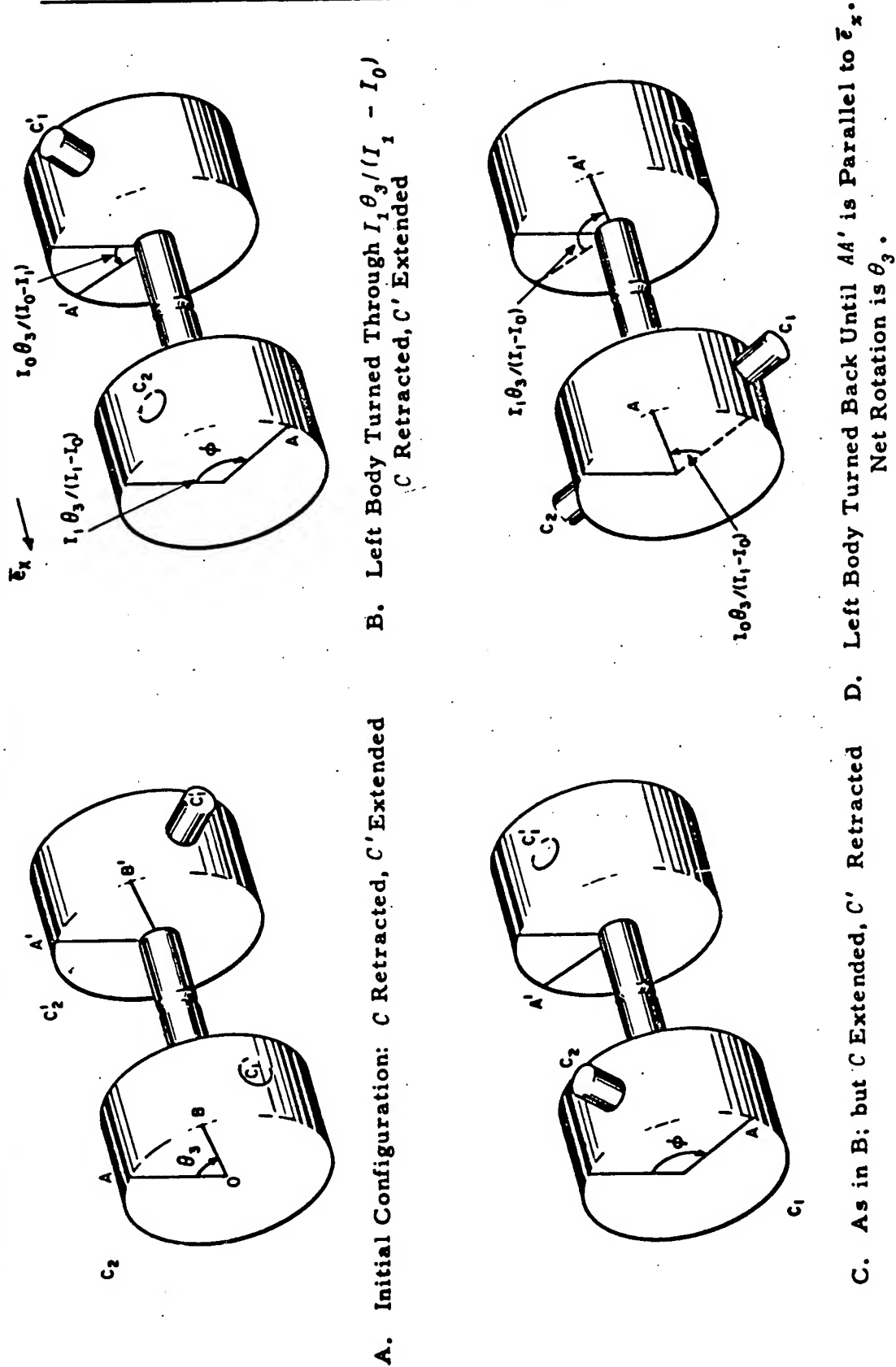
Fig. 24.   Schematic Diagram Showing Control by Changing Moment of Inertia

change of moment of inertia probably can not be accomplished merely by shifting locations of active components in the vehicle, the physical counterparts of the symbolic bosses represent dead weight.

Radiation from the Vehicle

A method of control can be based on the inertial reaction of radiation from the vehicle. It has been mentioned that about 6 kw are radiated continuously. If the center of radiating area can be located at a sufficient distance from the center of mass of the vehicle, an appreciable torque may be obtained. For example, if this distance is 5 m (an extreme case), the torque is of the order of 10^{-4} newton-meter (1000 dyne-cm). If this torque is not sufficient, the radiated energy can be supplemented deliberately. However, in order to use this effect as a control means, it is necessary to change either the location of the radiating area or the amount of radiation from it. Both alternatives are likely to be impossible, or at least mechanically impractical.

An alternate realization of the same effect can be visualized in electric light bulbs (or similar radiators) mounted in reflectors suspended on arms extending from the vehicle. By this means, a continuous fine control of torque for all attitudes can be maintained. However, by the considerations above, it may be expected that 5 to 6 kw of power will be required for each 10^{-4} newton-meter of control torque. As the perturbation torques on the vehicle are likely to be of the order of 10^{-4} to 10^{-3} newton-meter, this power requirement is patently too large for satellite operation.

Jet Reaction Thrust

An obvious means of attitude control is by jet reaction thrusts. If any mass is ejected from the vehicle with a certain momentum, then an opposite momentum is imparted to the vehicle by the reaction forces. Solid objects, liquids, or gases may be ejected. In the last case, the gas may be stored under pressure, formed by chemical reaction from solids or liquids, or liquid vaporized by heat.

The gas jets recommend themselves by the high velocity, and therefore high momentum per unit mass of ejectable material, which can be realized. Of the methods mentioned for obtaining the gas, the last seems preferable. Pressurizing the entire load of gas requires massive tanks. Chemical reactions are difficult or impossible to control as functions of the sensed attitude errors. In the case of vaporization of liquids, however, the waste heat from the orbital power source may be used, and only a small quantity of gas at any one time pressurized.

Some detailed comments about a system of this kind are presented in Chapter 9. Certain disadvantages are apparent at once. The small thrusts required have to be realized by subminiature mechanical components, which may be difficult to fabricate and operate. (For example, expansion of the gas to essentially zero pressure may freeze the controls and thus obstruct the orifices.) More important, perhaps, is the fact that a continuous control of these components as a function of attitude errors and error rates probably can not be effected. The systems are limited to off-on operation at a preas-signed constant thrust level. This implies that oscillations can not be damped out by the system, and that some attitude errors at all times must be tolerated.

On the other hand, a system of this kind might be made simple and re-liable. Therefore, it should not be rejected out of hand as a possible control system.

7.3. CONTROL BY REACTION WITH AMBIENT FIELDS

Gravitational Field of Earth

By lumping the mass of the vehicle into equivalent point masses lying along the vehicle forward axis, it was shown (section 2.3) that the normal vehicle attitude is unstable in the gravitational field of the earth. However, suppose that the vehicle were designed with its long axis normally in the zenith rather than in the forward direction. Then exactly the same considerations show that the gravitational field of the earth exerts a stabilizing torque rather than a perturbation torque on the vehicle. This kind of design, spoken of as a "stable attitude configuration," is of great importance in the control of satel-lite attitude.

The advantage of the stable attitude configuration is apparent. If the perturbation torque of section 2.3 becomes a restoring torque, the effective equation of motion of the vehicle for small pitch angular deviations θ_2 is

$$J\ddot{\theta}_2 + C\theta_2 = L \tag{63}$$

where C is the restoring "spring constant," J is the moment of inertia of the vehicle about the pitch axis, and L is the perturbation torque about this axis. Thus, for perturbation torques which are not too large, the vehicle undergoes undamped (but not unstable) vibrations about the equilibrium position. The plane of the angle θ_2 generally rotates about the zenith because of changing ratios of pitch and roll torques. The static angle of inclination under the torque L can be estimated numerically by supposing $C = 2 \times 10^{-3}$ newton-meter/radian, as in section 2.3, and $L = 10^{-4}$ newton-meter (1000 dyne-cm). The corres-ponding static angle would be $\theta_2 = 50$ milliradians. However, this could be re-duced by a greater elongation of the vehicle in a zenith direction.

Thus, the use of a stable attitude configuration limits angular deviations from the vertical caused by persistent torques. It also tends to limit the initial transient under torque impulses (from meteorite hits, for example).

There are disadvantages to the method, however. If the maximum perturbation torques have been underestimated, the restoring torque available to overcome them completely (i.e., to limit the angular deviations between small preassigned bounds) is not as large as might be wished. What is worse, from an operational point of view, is that there is no way in which the motion can be damped by this control method. Any perturbation torque or impulse is reflected in a nondecreasing oscillation of the vertical. Therefore, the method probably should be regarded as a supplement to some method which permits fine control and damping, rather than as an exclusive method for controlling the vertical. As such, it is of considerable value in enabling the operational requirements on the primary control system to be relaxed. For example, it is unnecessary to store enough energy to control against small persistent torques. It may be permissible, with this inherent vehicle stability, to sense attitude deviations only over a portion of the orbit (e.g., in daylight) and trust that the attitude deviation during the time when no sensing is done does not become too great.

In constructing a vehicle with a stable attitude configuration, there are a number of practical questions which must be considered. Some of these relate to provisions for heat dissipation from the skin, the elongation of the vehicle from the compact configuration used during the trajectory to the stable configuration, and the rotation of the vehicle into the stable attitude once its trajectory is established. Some of these considerations, together with the details of a possible physical design, are discussed in a supplementary report (Ref. 23).

Magnetic Field of Earth

If a loop of wire is rigidly attached to the vehicle, and a current sent through it, the resulting magnetic field will interact with the magnetic field of the earth. A torque will be exerted on the vehicle by the earth, and the vehicle's attitude will change in consequence. Thus, one has the basis for a possible attitude control system which utilizes the magnetic field of the earth for its controlling torques.

The first question to be decided is whether controlling torques for all attitudes can be obtained in this manner.

Let x, y, z be principal vehicle axes, as in section 3.4. Suppose that a coil of wire of directed area $\overline{A}$ and N turns is fixed in the vehicle. Suppose, further, that at some instant the vehicle is in a magnetic field whose magnetic

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induction vector is $\bar{B}$. If the coil is energized by a current i, the interaction of the field of the coil and the ambient magnetic field produces a torque $\bar{L}$ on the coil (and, therefore, on the vehicle) given by

$$\bar{L} = iN\bar{A} \times \bar{B} \tag{64}$$

No matter what the coil configuration or current magnitude, this vector equation for $\bar{L}$ implies that the torque is necessarily in a plane normal to $\bar{B}$. Clearly, no torque can be applied about an arbitrary vehicle-fixed axis, such as the axis of roll, pitch, or yaw, unless this axis happens to be perpendicular to $\bar{B}$.

Thus, only two possibilities remain for the generation of a control torque by magnetic means in the stated manner. Either a combination torque (roll and pitch, roll and yaw, pitch and yaw) must be accepted continuously; or, if torque purely about one axis is desired, only intermittent control at the instants when the field vector becomes perpendicular to the desired torque axis can be obtained.

Intermittent control of this kind can occur only as the vehicle crosses the magnetic equator and the normal magnetic meridian at its north and south apexes. In these two cases respectively, yaw and roll torques can be obtained during sufficiently short time intervals. The current for control can be initiated by flux gates which detect the vanishing of the yaw-axis and roll-axis components of the earth's field.

A pure pitch torque by this method is impossible, but an average pitch torque over one orbital period can be obtained at the expense of introducing possibly unwanted roll or yaw torques during this time. However, no net roll or yaw impulse need be imparted. Suppose that a pitch torque is desired, on the average, and that a roll torque can be tolerated so long as its average is zero over an orbital period.

The cone described by the magnetic field vector relative to the vehicle is depicted in Fig. 25. During a half-orbital period, there is an instant when it has the position $\bar{B}$, and another when it has the position $\bar{B}'$, which makes the same angle with the rear direction as $\bar{B}$ makes with the forward direction. The coil currents can be chosen in such a way that the control torques at these instants are $\bar{L}$ and $\bar{L}'$ respectively by merely reversing the direction of the current loop normal to the z-direction. The sum of these torques has a net pitch component (along the y-axis), but no roll component (along the x-axis). Because this is true for every such pair of $\bar{B}$-values, with a current reversal as the x-component of $\bar{B}$ goes to zero, the torque average over a half-orbital period (and similarly over the following half-period) is purely a pitch torque.

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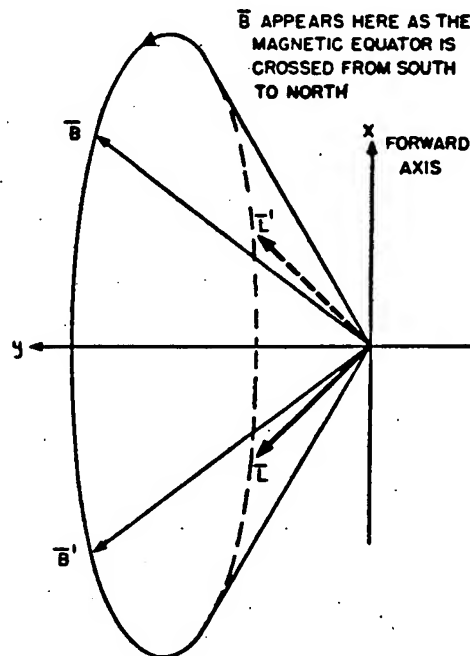
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Fig. 25. A Net Pitch Control Torque by
Magnetic Field of Earth

Aside from the possible direction of applied torques, there is the very
important question of the possible torque magnitudes which can be realized by
this method. At an altitude of 640 km (400 miles), the field of the earth is
$|\bar{B}| = 0.28$ gauss $= 2.8 \times 10^{-5}$ weber/sq m. For a circular coil of 0.5-m dia
the maximum torque amplitude which can be obtained per ampere turn is
$|\bar{L}| = 0.55 \times 10^{-5}$ newton-meter/ampere-turn. Because 10^{3} ampere turns is
not unrealistic, a torque of the order of 0.5×10^{-2} newton-meter (50,000 dyne-
cm) appears to be obtainable. This should be ample for the projected appli-
cation. For a coil of No. 10 B & S annealed copper wire, the power con-
sumption for this torque magnitude is about 40 w. It must be remembered,
however, in estimating both the adequacy of the torque and the magnitude of
the power consumption, that this control system operates for only a relatively
small fraction of the total orbital period.

A principal disadvantage of this magnetic method of control is that the
fields produced in the vehicle by the current loop may interfere with the oper-
ation of other electrical equipment. In particular, this method may entail mal-
function of the television equipment. The flux-gate sensing required to de-
termine the proper times for intermittent control is an added complexity. The
method does not appear promising compared with other possibilities developed
in the report.

Atmospheric Pressure Field

It has been mentioned in section 2.3 that the atmospheric pressure is too low for aerodynamic forces to be significant in attitude perturbation. It may be suspected that their controlling possibilities are also insignificant. Control (accompanied by some drag) may be obtained by projecting a large flat plate normally into the airstream. The maximum pressure on the plate occurs when the reflection of air molecules from the plate is specular. If the plate velocity of motion v is much greater than the expected molecular velocity c_m, and if ρ is the air density, the pressure acting on the surface for specular reflection is given by a formula of Tsien (Ref. 24) as $\rho(c_m^2 + 2v^2)$. It should be emphasized that this formula embodies several approximations, each tending to cause an overestimation of the pressure.

From Ref. 25, $\rho = 0.37 \times 10^{-11}$ kg/cu m, $c_m = 1.2 \times 10^3$ m/sec, $v = 0.76 \times 10^4$ m/sec (for 500 km altitude), whence the pressure is of the order of 5×10^{-6} newton/sq m (5×10^{-5} dyne/sq cm). Even with two control surfaces, each having an area of 10 sq m and its centroid 3 m from the vehicle center of mass, the maximum control torque obtainable by this method is of the order of 3×10^{-4} newton-meter (3000 dyne-cm). This amount of control torque is at best, marginal. Using later information of the upper atmosphere (Ref. 8), this estimate of control torque must be revised downward to about 3×10^{-7} newton-meter (3 dyne-cm). As the pressure given by $\rho(c_m^2 + 2v^2)$ is undoubtedly too high to begin with, this possibility of control must be rejected.

Additional difficulties with the method may be mentioned anticlimactically as the mechanical complexity required for adjustable control, and the critical limitation on the altitude of satellite operation. It should be noted, however, that the method cannot be rejected on the grounds of its concomitant energy drain by aerodynamic drag.

Radiation Incident on the Vehicle

A principal of allowing incident radiation from the sun to apply controlling torques to the vehicle is conceivable. A system of shutters of black and white panels can be so arranged that the ratio of black to white areas on the two sides of a principal axis is changed. The radiation, in being reflected more from the lighter side than from the darker side of this axis, applies a greater force to the former and supplies an overturning torque about the axis.

The details of such a method can be analyzed in the same way as the perturbation torque caused by incident radiation was analyzed in section 2.3. However, three major objections exist to the use of such a system, and a detailed discussion does not seem necessary. Most important is that the heat dissipation problem in the satellite is critical enough that incoming radiation through the black panels undoubtedly would be intolerable. A second objection is to the mechanical complexity required by such a system. Finally, but not least important, is that it does not seem possible to realize more than a few hundred dyne-centimeters of torque in this fashion.

8. CONTROL BY ACCELERATION WHEELS

8.1. PROBLEMS

As discussed in section 7.2, it is possible to obtain a control torque by means of three acceleration wheels mounted in the vehicle.

There are only two serious problems which must be discussed in connection with such a system. First, can sufficient momentum be stored by these wheels to provide a control torque adequate to compensate all expected perturbation torques? Second, what are the control equations which should be used to determine the required control torque in terms of the sensed attitude deviation?

It is shown in the sequel that wheels of reasonable maximum angular velocity and size have sufficient momentum storage capacity to provide the required controlling torques in roll and yaw. This is in part due to the fact that there are no persistent torques acting to perturb these attitudes, so that the average required control angular momentum over a long period of time is zero. Thus, only the total torque impulse over half of an orbital revolution need be stored at any one time, together with whatever impulse is estimated to correct for meteorite collisions over the course of a year. On the other hand, in pitch there may be small persistent torques whose control requires a constant wheel acceleration over an entire year, with a consequent excessive wheel velocity. Therefore, it is not certain (in the absence of very precise knowledge of such torques) whether an acceleration wheel can be used alone for pitch control. However, if a stable attitude configuration such as described in section 7.3 is used, any such constant torques in either pitch or roll will manifest themselves merely as a slight cocking of the vehicle against the restoring spring force of gravity. If this cocked position is made the zero of the sensing instrument, or if it is within a dead zone of the instrument, the acceleration wheel control system will not be called upon to counter this perturbation, and only the periodic torques will act against the control. In this case, it will appear feasible to use acceleration wheel control even in the presence of constant perturbation torques.

The question of the stability of the control system under various choices of control equation requires careful consideration. In particular, it will be shown that the control equations suggested for convenience in their mechanization (Ref. 1) will lead to undamped flywheel oscillations.

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8.2. EQUATIONS OF MOTION

For the Vehicle

It is convenient to write these equations out in some detail, in order to have explicit moment equations available, and to simplify only at the last stage. The equations of motion for the vehicle can be obtained directly from Eq. 11 to 13, which were derived for the motion with the yaw gyro. However, in the present case it is convenient to have them in a somewhat different form.

Denote by Ω_x, Ω_y, and Ω_z the components with respect to the principal vehicle-fixed axes of the angular velocity of the vehicle relative to inertial space. Let $\bar{P}$ be the perturbation torque on the vehicle, and $\bar{H}_x$, $\bar{H}_y$, and $\bar{H}_z$ the moments applied to the vehicle by the roll, pitch and yaw control wheels respectively. Then if the vehicle moments of inertia are I_v, J_v, and K_v about the x, y, and z axes respectively, as in Chapter 4, the Euler equations may be written

$$I_v \dot{\Omega}_x + (K_v - J_v)\Omega_y\Omega_z = P_x + (H_x)_x + (H_y)_x + (H_z)_x \qquad (65)$$

$$J_v \dot{\Omega}_y + (I_v - K_v)\Omega_x\Omega_z = P_y + (H_x)_y + (H_y)_y + (H_z)_y \qquad (66)$$

$$K_v \dot{\Omega}_z + (J_v - I_v)\Omega_x\Omega_y = P_z + (H_x)_z + (H_y)_z + (H_z)_z \qquad (67)$$

For the Acceleration Wheels

Designate variables pertaining to the roll, pitch, yaw by subscripts x, y, and z respectively, as in the case of the moments $\bar{H}$ of Eq. 65 to 67. Let I, J, and K, with appropriate subscripts, refer to wheel moments of inertia relative to principal vehicle axes, and let $\dot{\psi}$ refer to their spin angular velocities. Consider that the axis of rotation of the roll control wheel is constrained to lie along the vehicle x-axis, the pitch wheel axis along the y-axis, and the yaw wheel axis along the z-axis.

The angular velocity of the roll flywheel is $(\Omega_x + \dot{\psi})\bar{e}_x + \Omega_y\bar{e}_y + \Omega_z\bar{e}_z$ while the angular velocity of the coordinate axis in space is just the vehicle angular velocity. The angular momentum of this flywheel (using wheel symmetry properties) is

$$I_x(\Omega_x + \dot{\psi}_x)\bar{e}_x + J_x\Omega_y\bar{e}_y + J_x\Omega_z\bar{e}_z \qquad (68)$$

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It follows easily that its equations of motion are

$$I_x'(\ddot{\psi}_x + \dot{\Omega}_x) = - (M_x)_x \qquad (69)$$

$$J_x\dot{\Omega}_y + (I_x - J_x)\Omega_x\Omega_z + I_x\dot{\psi}_x\Omega_z = -(M_x)_y \qquad (70)$$

$$J_x\dot{\Omega}_z + (J_x - I_x)\Omega_x\Omega_y - I_x\dot{\psi}_x\Omega_y = -(M_x)_z \qquad (71)$$

Similarly, the equations of motion for the pitch wheel are

$$I_y\dot{\Omega}_x + (I_y - J_y)\Omega_y\Omega_z - J_y\dot{\psi}_y\Omega_z = -(M_y)_x \qquad (72)$$

$$J_y(\ddot{\psi}_y + \dot{\Omega}_y) = - (M_y)_y \qquad (73)$$

$$I_y\dot{\Omega}_z + (J_y - I_y)\Omega_x\Omega_z + J_y\dot{\psi}_y\Omega_x = -(M_y)_z \qquad (74)$$

while those for the yaw wheel are

$$I_z\dot{\Omega}_x + (K_z - I_z)\Omega_x\Omega_y + K_z\dot{\psi}_z\Omega_y = -(M_z)_x \qquad (75)$$

$$I_z\dot{\Omega}_y + (I_z - K_z)\Omega_x\Omega_z - K_z\dot{\psi}_z\Omega_x = -(M_z)_y \qquad (76)$$

$$K_z(\ddot{\psi}_z + \dot{\Omega}_z) = - (M_z)_z \qquad (77)$$

A Reduced Set

Equation 69 to 77 can be used with Eq. 65 to 67 to eliminate the unknown interaction moments between acceleration wheels and vehicle, to obtain

$$I_x\ddot{\psi}_x + I\dot{\Omega}_x + (K - J)\,\Omega_y\Omega_z - J_y\dot{\psi}_y\Omega_z + K_z\dot{\psi}_z\Omega_y = P_x \qquad (78)$$

$$J_y\ddot{\psi}_y + J\dot{\Omega}_y + (I - K)\,\Omega_x\Omega_z + I_x\dot{\psi}_x\Omega_z - K_z\dot{\psi}_z\Omega_x = P_y \qquad (79)$$

$$K_z\ddot{\psi}_z + K\dot{\Omega}_z + (J - I)\,\Omega_x\Omega_y - I_x\dot{\psi}_x\Omega_y + J_y\dot{\psi}_y\Omega_x = P_z \qquad (80)$$

where I, J, and K are the total moments of inertia of the vehicle and acceleration wheels

The wheel spin angular momenta are

$$H_x = I_x(\dot{\psi}_x + \Omega_x) \qquad (81)$$

$$H_y = J_y(\dot{\psi}_y + \Omega_y) \qquad (82)$$

$$H_z = K_z(\dot{\psi}_z + \Omega_z) \qquad (83)$$

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Using the components of $\bar{\Omega}$ implied by Eq. 9, the fact that $I \gg I_x$, $J \gg J_y$, $K \gg K_z$, and presuming that the three control wheels are identical $(I_x = J_y = K_z)$, the equations of motion become

$$I\ddot{\theta}_3 + [(I+K-J)\omega_2 - H_y]\dot{\theta}_1 + H_z\dot{\theta}_2 + (I\dot{\omega}_2 - \omega_1 H_z)\theta_1 + [(K-J)\omega_2$$

$$- H_y](\omega_1\theta_2 - \omega_2\theta_3) + [\dot{H}_x + \omega_2 H_z + I\dot{\omega}_1] = P_x \tag{84}$$

$$J\ddot{\theta}_2 + [(I-K-J)\omega_1 + H_x]\dot{\theta}_1 - H_z\dot{\theta}_3 - (J\dot{\omega}_1 + H_z\omega_2)\,\theta_1 + [(I-K)\omega_1$$

$$+ H_x](\omega_1\theta_2 - \omega_2\theta_3) + [\dot{H}_y - \omega_1 H_z + J\dot{\omega}_2] = P_y \tag{85}$$

$$K\ddot{\theta}_1 + [(J-I+K)\omega_1 - H_x]\,\dot{\theta}_2 + [(J-I-K)\omega_2 + H_y]\,\dot{\theta}_3 + [(J-I)(\omega_2{}^2 - \omega_1{}^2)$$

$$+ H_x\omega_1 + H_y\omega_2]\theta_1 + K\dot{\omega}_1\theta_2 - K\dot{\omega}_2\theta_3 + [(J-I)\omega_1\omega_2 - H_x\omega_2$$

$$+ H_y\omega_1 + \dot{H}_z] = P_z \tag{86}$$

This is a set of equations for the angular motions of the vehicle away from the local trihedral, given the wheel angular momenta and the orbit characteristics.

However, these are not yet in linearized form. The acceleration wheel angular momenta are to be determined by the control equations, and will be proportional to the sensed attitude deviations and deviation rates. Therefore, any products of the form $H\theta$ and $H\dot{\theta}$ should be discarded in the same way that cross products and powers of θ and $\dot{\theta}$ were previously discarded. This leaves the linearized set

$$I\ddot{\theta}_3 + (I+K-J)\omega_2\dot{\theta}_1 + I\dot{\omega}_2\theta_1 + (K-J)\omega_2(\omega_1\theta_2 - \omega_2\theta_3)$$

$$+ [\dot{H}_x + \omega_2 H_z + I\dot{\omega}_1] = P_x \tag{87}$$

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$$J\ddot{\theta}_2 + (I-K-J)\omega_1\dot{\theta}_1 - J\dot{\omega}_1\theta_1 + (I-K)\omega_1(\omega_1\theta_2 - \omega_2\theta_3)$$

$$+ [\dot{H}_y - \omega_1 H_z + J\dot{\omega}_2] = P_y \tag{88}$$

$$K\ddot{\theta}_1 + (J-I+K)\omega_1\dot{\theta}_2 + (J-I-K)\omega_2\dot{\theta}_3 + (J-I)(\omega_2{}^2 - \omega_1{}'^2)\theta_1 + K\dot{\omega}_1\theta_2$$

$$- K\dot{\omega}_2\theta_3 + [(J-I)\omega_1\omega_2 - H_x\omega_2 + H_y\omega_1 + \dot{H}_z] = P_z \tag{89}$$

Restriction on the Problem

In the special case $\omega_1 = \dot{\omega}_2 = 0$, that of a plane circular orbit, Eq. 87 to 89 take the simple form

$$I\ddot{\theta}_3 + (I+K-J)\omega_2\dot{\theta}_1 + (J-K)\omega_2{}^2\theta_3 + (\dot{H}_x + \omega_2 H_z) = \Pi_x \tag{90}$$

$$J\ddot{\theta}_2 + \dot{H}_y = \Pi_y \tag{91}$$

$$K\ddot{\theta}_1 + (J-I-K)\omega_2\dot{\theta}_3 + (J-I)\omega_2{}^2\theta_1 + (\dot{H}_z - \omega_2 H_x) = \Pi_z \tag{92}$$

For the remainder of this preliminary study it will be supposed that this case maintains, recognizing that the orbital perturbation torques discussed in section 2.2 are not thereby taken into account. However, there is no loss in generality if these and other neglected terms are supposed to be absorbed with the perturbation torque $\bar{P}$ into the equivalent perturbation torque $\bar{\Pi}$.

A second basic restriction will be added, in adjoining a set of control equations to the equations of motion (Eq. 90 to 92). It will be assumed that there are no sensing errors, so that it is possible to calculate control torques from a perfect knowledge of the instantaneous roll, pitch, and yaw deviations. Ultimately, it will be necessary to carry out an error analysis in which the interaction of sensing and control errors is permitted. However, it does not seem necessary to do this in investigating the general adequacy of certain kinds of control equations.

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8.3. CONTROL EQUATIONS

Pitch Control

It is clear from Eq. 90 to 92 that pitch control can be accomplished, in the first approximation, by the pitch acceleration wheel alone. The behavior of this wheel (in the form of H_y) does not enter the equations of either roll or yaw behavior, and there is no coupling between these and the pitch control system. Therefore, it suffices for pitch control to sense the pitch deviation angle and its rate and to adjust H_y according to

$$\dot{H}_y = - B_y \dot{\theta}_2 - F_y \theta_2 \tag{93}$$

This expression is just the torque to be applied to the pitch acceleration wheel motor, and can be made proportional to applied voltage for a properly designed motor.

A Conditionally Stable Control

Because there is coupling between the roll and yaw wheel momenta and the corresponding vehicle motions, these two controls must be considered together.

Using equations developed earlier in the chapter for the moments applied to the acceleration wheels, it can be shown that in the notation of this report and in linearized form, the control equations suggested in Ref. 1 can be written

$$\dot{H}_x = B_x \dot{\theta}_3 + F_x \theta_3 + \left(\frac{I_z - I_z}{I_z} \right) \omega_2 H_z \tag{94}$$

$$\dot{H}_z = B_z \dot{\theta}_1 + F_z \theta_1 + \left(\frac{I_x - J_x}{I_x} \right) \omega_2 H_x \tag{95}$$

If the roll and yaw flywheels are identical and κ is defined by

$$\kappa^2 = \frac{I_z - I_z}{I_z} = \frac{I_x - J_x}{I_x} < 1 \tag{96}$$

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Equations 94 and 95 can be rewritten

$$\dot{H}_x + \kappa^2 \omega_2 H_z = B_x \dot{\theta}_3 + F_x \theta_3 \tag{97}$$

$$\dot{H}_z - \kappa^2 \omega_2 H_x = B_z \dot{\theta}_1 + F_z \theta_1 \tag{98}$$

These, together with Eq. 90 and 92, completely determine the motion of the vehicle.

It is possible to eliminate H_x and H_z from these equations, obtaining either a pair in θ_1 and θ_3 or a single equation in θ_1 or θ_3. However, the flywheel motion can be inferred without solving these equations. For, differentiating either of Eq. 97 and 98 and using the other, one obtains the typical equation

$$\ddot{H} + \kappa^4 \omega_2^2 H = \text{terms in } \theta_1, \theta_3 \text{ and their derivatives} \tag{99}$$

No matter what the θ_1 and θ_3 behavior, whether stable, conditionally stable, or unstable, it is evident from Eq. 99 that an undamped oscillation of both wheels generally will occur after any transients have died out. Thus, the motion of the acceleration wheels themselves may be considered to be only conditionally stable, which is not desirable from a design standpoint.

A Stable Control

It seems possible to obtain a stable control, with a damped wheel oscillation, by using control equations of the form

$$\dot{H}_x = B_x \dot{\theta}_3 + F_x \theta_3 - \kappa^2 \omega_2 H_z - G_x H_x \tag{100}$$

$$\dot{H}_z = B_z \dot{\theta}_1 + F_z \theta_1 + \kappa^2 \omega_2 H_x - G_z H_z \tag{101}$$

The system transfer function can be obtained from Eq. 90, 92, 100, and 101 by a rather tedious manipulation. The characteristic equation is

$$
\begin{vmatrix}
Is^2 + (J-K)\omega_2^2 & (I+K-J)\omega_2 s & s & \omega_2 \\
-(I+K-J)\omega_2 s & Ks^2 + (J-I)\omega_2^2 & -\omega_2 & s \\
-(B_x s + F_x) & 0 & s+G_x & \kappa^2 \omega_2 \\
0 & -(B_z s + F_z) & -\kappa^2 \omega_2 & s+G_z
\end{vmatrix} = 0 \qquad (102)
$$

Its roots have not been investigated for all possible parameter values, but it can be shown that a subjectively reasonable set of parameter values corresponds to system stability.

Further investigations are required of the coupling of the sensing and control elements through the control equations. Attempts to determine optimum coupling form, or at least optimum coupling coefficients, are of considerable interest. It must be remembered that the computer mechanization for such a set of control equations may be one of the principal problems connected with this method of control.

An Illustrative Example

Suppose that for a stable attitude configuration, $I = J = 10^3$ kg-m^2 and $K = 10^2$ kg-m^2. Some possible control parameters (not necessarily optimum) are

$$F_x/I = 10^{-4} \text{ radian/sec}^2$$

$$F_z/K = 10^{-2} \text{ radian/sec}^2$$

$$B_x/I = 2 \times 10^{-2} \text{ radian/sec}^2$$

$$B_z/K = 2 \times 10^{-1} \text{ radian/sec}$$

$$G_x = 0$$

$$G_z = 10^{-2} \text{ sec}^{-1}$$

$$\kappa^2 = 0.5$$

These values imply that the characteristic equation is

$$[s + 0.137][s + 0.073][s^2 + 0.02s + 10^{-4}][s^2 + 3.6 \times 10^{-7} s + 0.965 \times 10^{-6}] = 0 \qquad (103)$$

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For this case, there are two damped nonoscillatory modes and two damped oscillatory modes. One of the latter has a characteristic frequency about ten times orbital frequency, and critical damping. The second oscillatory mode has close to orbital frequency and very small damping. Therefore, this is not a very desirable control system, as it will have relatively large response to excitations having orbital frequency. However, it is stable.

A notion of the momentum storage requirements of the wheels can be obtained by supposing that constant perturbation torques $\Pi_x = \Pi_y = 10^{-3}$ newton-meter act on the vehicle. Then from Eq. 90, 92, 100 and 101 it is easy to show for the example at hand that the following steady state values maintain:

$$H_x \approx H_z \approx 1 \text{ kg-m}^2/\text{sec} \quad (10^7 \text{ gm-cm}^2/\text{sec})$$

$$\theta_3 \approx \qquad 5 \text{ milliradians}$$

$$\theta_1 \approx \qquad 10 \text{ milliradians}$$

These values, though high, are not intolerable. Furthermore, they can be reduced by a more nearly optimum choice of control parameters or by a less conservative (and possibly more realistic) estimate of the perturbation torques.

The wheel design for the system described is influenced by such important factors as

1. The adverse affect on heat transfer of the low atmospheric pressure, unless the system is subjected to some environmental control.
2. Evaporation of lubricants and impregnants at low pressure and high temperatures.
3. The absence of gravitational forces, which reduces bearing loads.
4. The requirement for operational life of one year.

A compromise must be made between wheel weight and spin velocity. Small wheels at high spin velocity will have increased probability of spin bearing failure, whereas heavier wheels may exceed weight and space requirements and will place large loads on their supports during the trajectory acceleration phase. If the dimensional relationships of the wheel of the NAA Mark 1 gyro are maintained, 8-kg wheels running at 2000 rpm will satisfy the maximum angular momentum requirements of the example.

9. CONTROL BY JET THRUST

9.1. CLASSIFICATION OF METHODS

As mentioned in section 7.2, of the various possibilities which exist for realizing control torques from reaction thrusts the most promising seems to be that in which vapor formed by heating a liquid is ejected from the vehicle in a jet. The purpose of this chapter is to describe two possible mechanizations of such a jet system and to discuss their feasibilities and disadvantages.

It is clear that six jets are required for the control of all three attitudes, as two jets are necessary to produce a pure couple (without translation-provoking unbalanced forces) about each control axis. However, for convenience, only one jet is represented in the system description below.

The working fluid for the systems described is steam, which seems to have suitable physical properties, but further investigation of other fluids is desirable. The heat source is the waste heat from the orbital power plant.

9.2. MECHANIZATIONS

Two principles of operation seem obvious possibilities for realizing the jet steam. Their relative merits have not been investigated, so both are presented as typical systems worthy of further consideration.

The first system may be called a "repeated blowdown system" (Fig. 26). It meters a quantity of water stored at low pressure into a heat exchanger, which converts the water into high pressure superheated steam. The steam is collected in an accumulator and expanded as required through a supersonic nozzle to produce thrust. When the pressure in the accumulator drops below the water tank pressure, the metering valve admits more water into the heat exchanger, and the process repeats itself.

The second system may be called a "constant pressure system" (Fig. 27). It heats the water directly in its storage tank, producing steam in the tank space beyond the liquid surface. (Because of the nearly vanishing gravitational field, the liquid and vapor phases may have to be separated centrifugally.) A constant water temperature in the tank produces a constant vapor pressure in the tank, so that the thrust of the system is effectively constant.

Certain observations can be made at once on the difficulties and advantages of the two systems. The former produces an intermittent and probably somewhat variable thrust, which may complicate the problem of system control. It is not
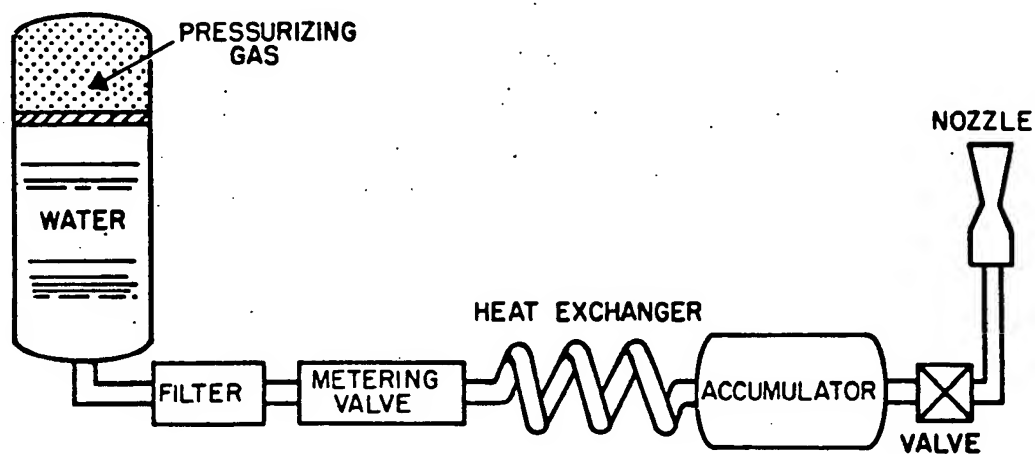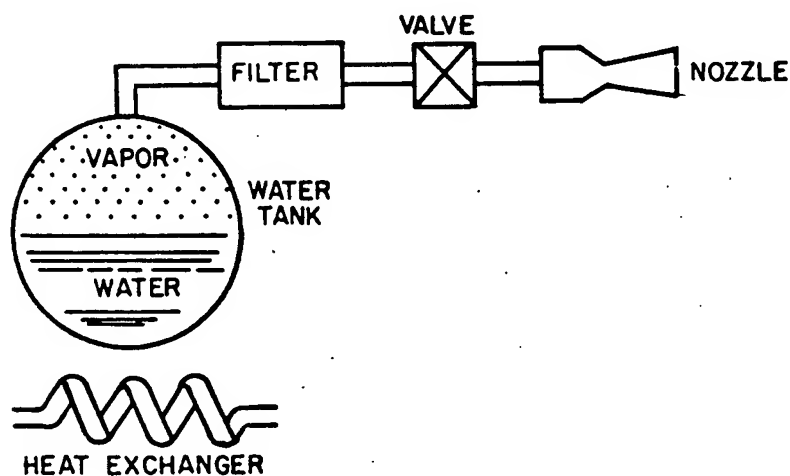
Fig. 26. Repeated Blowdown System

Fig. 27. Constant Pressure System

clear that a suitable metering valve for this system can be developed without some difficulty. On the other hand, while the second system may provide more uniform thrusts, it also requires a very large and sturdy tank for the pressurization of a quantity of water. The added weight of such a system may well be excessive.

Both systems are subject to the difficulties remarked in section 7.2, namely those associated with the miniature nature of the system, and those associated with the (more or less) constant thrust level, which prohibits a fine control.

9.3. DESIGN AND PERFORMANCE CONSIDERATIONS

Suppose that for the repeated blowdown system the steam is initially at 427 C (800 F) and 6.89×10^3 newton/sq m (500 lb/sq in.) and that it exhausts to -101 C (-150 F) and zero pressure. The total weight of water available is supposed to be 36.4 kg (80 lb), which is to last for one year. For illustrative purposes, a nozzle throat diameter of 2.54×10^{-5} m (0.001 in.) is chosen. Using the thermodynamic properties of steam, a number of parameters of interest can be calculated:

1. Flow rate through nozzle. Grashof's formula for the flow of superheated steam through an orifice (Ref. 26) is used. For the example under consideration, the flow rate is 2.03×10^{-6} kg/sec.

2. Jet velocity. Denoting by g the acceleration of gravity, by Q the mechanical equivalent of heat, and by Δh the enthalpy difference across the nozzle,

$$V_e = \sqrt{2gQ\Delta h} = 2100 \text{ m/sec } (6900 \text{ fps})$$

3. Thrust. The thrust force is equal to the product of mass rate and ejection velocity, or 4.3×10^{-3} newton per nozzle. The specific thrust per unit weight flow of vapor, is 216 sec.

4. Moment L. Assuming a moment arm of 1.5 m between a nozzle pair, the maximum control torque is $L = 6.45 \times 10^{-3}$ newton-meter.

5. Operating time at full thrust. If six nozzles are ejecting vapor and realizing the maximum thrust level continuously for a year, a fluid mass of 384 kg will be used. If the limit is 36.4 kg, the six nozzles will operate for only 9.5 percent of the year on orbit.

6. Maximum total impulse. Dividing the available fluid equally among the three attitudes, the total control impulse realized for each attitude at the above rate of ejection and torque level will be 3.9×10^4 newton-meter-sec.

7. Average torque for each control axis. On the average, over one year, the impulse above corresponds to a continuous torque of about 10^{-3} newton-meter.

8. Maximum required heat rate. With six nozzles operating simultaneously at mass rate G, the heat rate required to supply the superheated steam

from water is $6G(h_i - h_0)$, where h_0 is enthalpy per pound of water of 100 F and h_1 is enthalpy of the vapor at the nozzle inlet. For the conditions above, the requirement is 38 w (0.036 BTU/sec).

Similar characteristics may be given for the constant pressure system. However, this does not seem necessary in the present report. If such vapor jet systems were to be studied more thoroughly, some of the additional relations it would be desirable to find would be

1. Expelled mass as a function of total impulse for constant boiler pressure.
2. Uncertainty in controlled impulse as a function of the duration of a continuous impulse.
3. Thrust as a function of nozzle characteristics for constant boiler pressure.

NOMENCLATURE

$\overline{A}$ — vector normal to a plane coil, with length equal to area of coil

a — typical length parameter

B — coefficient of linear viscous damping term in $f(\beta,\dot{\beta})$

$\overline{B}$ — magnetic field vector

$B_x,\ B_y,\ B_z$ — damping coefficients for roll, pitch, and yaw acceleration wheels control

b — source brightness

C — typical constant (possibly with numerical subscript)

c — velocity of light

c_m — expected molecular velocity

D — diameter of optical system

d — diameter of sensitive cell

e_q — unit vector along axis of any variable q

F — coefficient of linear spring term in $f(\beta,\dot{\beta})$

$\mathcal{F}_1$ — first Fourier component of $\omega_z(t)$

f — focal length of optical system

$f(\beta,\dot{\beta})$ — coupling function between gyro and vehicle

G — universal constant of gravitation

g — acceleration of gravity at earth's surface

$H,\vec{H}$ — typical angular momentum. H_x, H_y, and H_z are defined by Eq. 81 to 83.

h — enthalpy

$I,\ J,\ K$ — total moment of inertia of vehicle and its internal rotating components about the x, y, and z axes respectively

I_0, I_1	typical moments of inertia
I_G, J_G, K_G	moment of inertia of yaw gyro about x'', y'', and z' axes respectively (including gimbal inertia, when supposed to exist, or inertia of rotating internal parts)
I_V, J_V, K_V	total moment of inertia about x, y, and z axes respectively of vehicle minus the rotating parts currently discussed
I_x, I_y, I_z	moment of inertia of roll, pitch, and yaw acceleration wheels respectively about vehicle x-axis
i	coil current
J_x, J_y, J_z	moment of inertia of roll, pitch, and yaw acceleration wheels respectively about vehicle y-axis
K_x, K_y, K_z	moment of inertia of roll, pitch, and yaw acceleration wheels respectively about vehicle z-axis
L	typical torque
$\vec{L}$	externally applied torque on vehicle arising from control system
$\vec{L}_{GR}$	torque applied to gyro by gimbal rings
$\vec{L}_{VR}$	torque applied to vehicle by gimbal rings
M	mass of earth
M_x, M_y, M_z	torque applied to vehicle by roll, pitch, and yaw acceleration wheels respectively
m	typical particle mass
N	number of turns of coil
$\vec{P}$	perturbation torques on yaw gyro
ΔP	differential pressure between pressure gauges
P_1	probability of meteorite hits
P_a	ambient pressure at altitude

$p(m)$	probability density distribution for meteorite hits
Q	mechanical equivalent of heat
R	average orbital radius
R_h	radius of earth at horizon
R_0	average earth radius
s	complex system frequency
t	time
V_e	gas exit velocity from jet
v	path speed of vehicle
W_a	apparent weight of particle in satellite
W_e	earth weight of particle
x, y, z	principal body-fixed axes: roll, pitch, and yaw respectively
x'', y'', z'	set of axes related to x, y, z axes by the transformation of Fig. 14.
α	angle of rotation of gyro about intermediate gimbal axis (Fig. 14)
α_0	sermivertex angle of sun's cone
α_1	elevation of sun or star above horizon
β	angle of rotation of outer gimbal ring relative to the plane of the forward and zenith vehicle-fixed axis (Fig. 14)
γ	orbital inclination to equator
δ	inclination error in determining vertical
ϵ	angle of dip of horizon
ζ, η, ξ	axes of local trihedral of orbit: negative normal, binormal, and tangent respectively

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θ_1	yaw angle defined by Fig. 12
θ_2	pitch angle defined by Fig. 12
θ_2^*	apparent pitch $(= \theta_2 + \delta)$
θ_3	roll angle defined by Fig. 12
κ	defined by Eq. 96
λ	an angle coordinate locating a pendulum suspension relative to the vehicle center of mass
$\Pi, \overline{\Pi}$	linear combinations of perturbation torque components and their derivatives
Π_1	a linear combination of Π and its derivatives
$1/\rho$	instantaneous orbital curvature
$1/\tau$	instantaneous orbital torsion
ϕ	defined by Eq. 31 as $\theta_1 + \alpha$
$\dot{\psi}_x, \dot{\psi}_y, \dot{\psi}_z$	spin angular velocities of roll, pitch, and yaw acceleration wheels respectively
Ω	spin axis of yaw gyro (Chapter 4) or of rotating parts (Chapter 7)
$\Omega_x, \Omega_y, \Omega_z$	components of $\bar{\omega}_\nu$ with respect to the x, y, and z axes
ω_ν	angular velocity of vehicle in space
ω_1	defined as v/τ
ω_2	defined as v/ρ

Dots over letters represent time derivatives.
Subscripts x, y, z, etc. identify vector components relative to axes indicated.

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